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Bayesian Dynamic Factor Analysis of a Simple Monetary DSGE Model

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Abstract

When estimating DSGE models, the number of observable economic variables is usually kept small, and it is conveniently assumed that DSGE model variables are perfectly measured by a single data series. Building upon Boivin and Giannoni (2006), we relax these two assumptions and estimate a fairly simple monetary DSGE model on a richer data set. Using post-1983 U.S. data on real output, inflation, nominal interest rates, measures of inverse money velocity, and a large panel of informational series, we compare the data-rich DSGE model with the regular – few observables, perfect measurement – DSGE model in terms of deep parameter estimates, propagation of monetary policy and technology shocks and sources of business cycle fluctuations. We document that the data-rich DSGE model generates a higher implied duration of Calvo price contracts and a lower slope of the New Keynesian Phillips curve. To reduce the computational costs of the likelihood-based estimation, we employed a novel speedup as in Jungbacker and Koopman (2008) and achieved the time savings of 60 percent.

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I. INTRODUCTION

When estimating dynamic stochastic general equilibrium (DSGE) models, the number of observable economic variables is usually kept small, and for convenience it is assumed that the model variables are perfectly measured by a single – often quite arbitrarily selected – data series. In this paper, we relax these two assumptions and estimate a version of the monetary DSGE model with a standard New Keynesian core on a richer data set. Building upon Boivin and Giannoni (2006), this so called *data-rich DSGE model* can be seen as a combination of a regular DSGE model and a dynamic factor model in which factors are the economic state variables of the DSGE model and the transition of factors is governed by a DSGE model solution.

We use the post-1983 U.S. data on real output, inflation, nominal interest rates, measures of inverse money velocity and a large panel of the other informational macroeconomic and financial series compiled by Stock and Watson (2008) to estimate and compare the new data-rich DSGE model with a regular – few observables, perfect measurement – DSGE model, both sharing the same theoretical core. The estimation involves Bayesian Markov Chain Monte Carlo (MCMC) methods. Because of the data set's high panel dimension, the likelihood-based estimation of the data-rich DSGE model is computationally very challenging. To reduce the costs, we employed a novel speed-up as in Jungbacker and Koopman (2008) and achieved computational time savings of 60 percent.

We document that the data-rich DSGE model generates a higher duration of the Calvo price contracts and a lower implied slope of the New Keynesian Phillips curve measuring the elasticity of current inflation to real marginal costs. As we move from the regular to the data-rich DSGE model, we find that: (i) the role of technology innovations in generating fluctuations in real output, inflation and interest rates is noticeably reduced; and that (ii) the contribution of monetary policy shocks to cyclical fluctuations of the interest rates increases from 4 to 14-17 percent. Regarding dynamic propagation, we establish that (i) despite some slight on-impact differences, the responses of all primary observables (real GDP, GDP deflator inflation, fed funds rate and real M2) to the monetary policy innovation remain theoretically plausible and quantitatively close in the regular and in the data-rich DSGE models; and that (ii) the regular DSGE model tends to overestimate all effects of TFP shocks, though on impact they might not have been too different. Finally, we find some puzzling results for the responses of industrial production, the PCE deflator inflation and the CPI inflation to monetary tightening, which may indicate the potential misspecification of our theoretical DSGE model.

The paper is organized as follows. In Section II, we present a data-rich DSGE model with a New Keynesian core to be used in the subsequent empirical analysis. Our econometric methodology to estimate the data-rich DSGE model and also the Jungbacker-Koopman computational speed-up are discussed in Section III. Section IV describes our data set and transformations. In Section V we conduct the empirical analysis of the regular and the data-rich DSGE models. We begin by discussing the choice of the prior distributions of model parameters and then describe the posterior estimates of deep structural parameters in both models. Second, we compare the estimated DSGE state variables from our data-rich and from the regular DSGE model. Finally, we explore the differences that the regular and the data-rich DSGE models imply about the sources of business cycle fluctuations and about the propagation of structural innovations, notably the monetary policy and technology shocks, to the real output, inflation, interest rates and real money balances. Section VI concludes.

II. DATA-RICH DSGE MODEL

In this section, we begin by defining what we refer to as the data-rich DSGE model and contrast it with the regular DSGE model. Then, we present a fairly standard New Keynesian business cycle core that will be shared by both types of models.

In any DSGE model, economic agents solve intertemporal optimization problems built from explicit preferences and technology assumptions. Moreover, decision rules of these agents depend upon a number of exogenous stochastic disturbances that characterize uncertainty in the economic environment. The equilibrium dynamics of a DSGE model are captured by a system of non-linear expectational difference equations. The standard approach in the literature is to derive a log-linear approximation to this non-linear system around its deterministic steady state and then to solve numerically the resulting linear rational expectations system by one of the available methods.[1]

This numerical solution delivers a vector autoregressive process for S_t, the vector collecting all non-redundant state variables of the DSGE model, and a linear relationship between the remaining DSGE model variables z_t and the current state S_t:

$$z_t = \mathbf{D}(\boldsymbol{\theta})S_t \tag{1}$$

$$S_t = \mathbf{G}(\boldsymbol{\theta})S_{t-1} + \mathbf{H}(\boldsymbol{\theta})\varepsilon_t, \qquad \text{where } \varepsilon_t \sim iid\ N(0, \mathbf{Q}(\boldsymbol{\theta})). \tag{2}$$

The matrices in (1) and (2) are the functions of structural parameters $\boldsymbol{\theta}$ characterizing preferences and technology in a DSGE model. For convenience, we assume that the exogenous shocks ε_t are mean-zero normal random variables with diagonal covariance matrix $\mathbf{Q}(\boldsymbol{\theta})$. In what follows we will refer to S_t as the *DSGE model states* or the DSGE model state variables. We will also refer to the elements of $\overline{S}_t = [z_t', S_t']'$, the vector collecting all variables in a given DSGE model, as the DSGE model concepts or simply *model concepts*. The typical examples of model concepts could be inflation, output, technology shock, capital stock and so on. By definition of $\overline{S}_t$:

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[1] Please see Sims (2002), Blanchard and Kahn (1980), Klein (2000), Uhlig (1999), and King and Watson (2002).

$$\bar{S}_t = \begin{bmatrix} \mathbf{D}(\boldsymbol{\theta}) \\ \mathbf{I} \end{bmatrix} S_t \tag{3}$$

In order to estimate our DSGE model on a set of observables $X^T = [X_1, ..., X_T]'$, a state-space representation of the model is constructed by augmenting (1)-(2) with a number of measurement equations that connect model concepts in $\bar{S}_t$ to data indicators in vector X_t.

A. Regular vs. Data-Rich DSGE Models

Depending on the number of data indicators and on how we connect them to the model concepts, we will distinguish regular and data-rich DSGE models. In *regular* DSGE models, the number of observables contained in X_t is usually kept small (most often equal to the number of structural shocks) and model concepts are often assumed to be perfectly measured by a single data indicator.[2] For example, Lubik and Schorfheide (2004), in a DSGE model with three structural shocks, specify the following measurement equations for real output $\tilde{x}_t$, inflation $\tilde{\pi}_t$, and the nominal interest rate $\tilde{R}_t$ (we omit the intercept for simplicity):

$$\underbrace{\begin{bmatrix} \text{RealGDP}_t \\ \text{CPI\_Inflation}_t \\ \text{FedFundsRate}_t \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 4 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 4 & 0 & \cdots & 0 \end{bmatrix}}_{\mathbf{\Lambda}} \cdot \underbrace{\begin{bmatrix} \tilde{x}_t \\ \tilde{\pi}_t \\ \tilde{R}_t \\ \vdots \end{bmatrix}}_{\bar{S}_t} \tag{4}$$

Similarly, Smets and Wouters (2007) estimate a DSGE model with seven structural shocks on seven key U.S. macro variables: again assuming one-to-one model concept-data indicator correspondence and perfect measurement.

Following an important contribution of Boivin and Giannoni (2006), *data-rich* DSGE models relax these assumptions and allow for: (i) the presence of measurement errors or, alternatively, of terms capturing the theoretical gap between a particular data indicator and a model concept it is supposed to measure; (ii) multiple data indicators $X_{j,t}$ measuring the same model concept $\bar{S}_{i,t}$, and (iii) many informational data series in X_t with an unknown link to specific model concepts that load on all DSGE model states (and that may contain useful information about the state of the economy). We call the *core series* X_t^F the part of X_t in which each data indicator loads on a single model concept $\bar{S}_{i,t}$ only (although same $\bar{S}_{i,t}$ may have several data indicators measuring it):

[2] The underlying reason is to avoid the so-called stochastic singularity. The likelihood function for observables X_t with dimension exceeding the number of structural shocks will be degenerate, since according to DSGE model some X_t 's can be perfectly (deterministically) predicted from others and this is obviously not true in the data. The solution is to add measurement errors (or theoretical gaps between the model concept and the data indicator) as e.g. in Altug (1989), Sargent (1989), and Ireland (2004), or to add more shocks, e.g., as in Leeper and Sims (1994), and Adolfson, Laseen, Linde, Villani (2008).

$$X_t^F = \mathbf{\Lambda_F} \overline{S}_t + e_t^F \,, \tag{5}$$

where each row of $\mathbf{\Lambda_F}$ contains just one non-zero element. We call the *non-core series* X_t^S the remaining part of X_t that is not supposed to measure any model concept and therefore loads freely on all DSGE model states:

$$X_t^S = \mathbf{\Lambda_S} S_t + e_t^S \tag{6}$$

For example, in a simple closed-economy DSGE model of Lubik and Schorfheide (2004), the core series might have been various measures of real output (e.g., real GDP, industrial production), of inflation (e.g., CPI inflation, PCE deflator inflation) or of the nominal interest rate; the non-core series might include exchange rates, real exports and imports, stock returns and similar data indicators not related directly to any model concept. We partition $\mathbf{\Lambda_F} = \left[\mathbf{\Lambda}_{\mathbf{F},1} \mid \mathbf{\Lambda}_{\mathbf{F},2} \right]$ conformably and use definition (3) to obtain the measurement equation in the data-rich DSGE model for demeaned X_t:

$$\underbrace{\begin{bmatrix} X_t^F \\ \hline X_t^S \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} \mathbf{\Lambda}_{\mathbf{F},1}\mathbf{D(\theta)} + \mathbf{\Lambda}_{\mathbf{F},2} \\ \hline \mathbf{\Lambda_S} \end{bmatrix}}_{\mathbf{\Lambda(\theta)}} S_t + \underbrace{\begin{bmatrix} e_t^F \\ \hline e_t^S \end{bmatrix}}_{e_t}, \tag{7}$$

where the measurement errors e_t may be serially correlated, but uncorrelated across different data indicators ($\mathbf{\Psi}$, $\mathbf{R}$ are diagonal):

$$e_t = \mathbf{\Psi} e_{t-1} + v_t, \qquad v_t \sim iid\ N(\mathbf{0}, \mathbf{R}) \,. \tag{8}$$

So the state-space representation of the data-rich DSGE model consists of transition equation (2) and measurement equations (7)-(8).

B. Environment

In this paper, we use a relatively standard New Keynesian business cycle core that will be shared by the data-rich and the regular DSGE models. It features capital as the factor of production, nominal rigidities in price setting, and investment adjustment costs. The real money stock enters households' utility in additively separable fashion as in Walsh (2003, Ch. 5), and Sidrauski (1967). In terms of a specific version of the model, we draw upon the work of Aruoba and Schorfheide (2009) and their money-in-the-utility specification.

The economy is populated by households, final and intermediate goods-producing firms and a central bank (monetary authority). A representative household works, consumes, saves, holds money balances and accumulates capital. It consumes the final output manufactured by perfectly competitive final good firms. The final good producers produce by combining a continuum of differentiated intermediate goods supplied by monopolistically competitive intermediate goods firms. To manufacture their output, intermediate goods producers hire labor and capital services from households. Also, when optimizing their prices, intermediate goods firms face the nominal price rigidity a la Calvo (1983), and those firms that are unable to re-optimize may index their price to lagged inflation. Monetary policy is conducted by the central bank setting the one-period nominal interest rate on public debt via a Taylor-type interest rate feedback rule. Given the interest rate, the central bank supplies enough nominal money balances to meet equilibrium demand from households.

Our DSGE model is more elaborate than the basic three-equation model used in Woodford (2003), but is "lighter" than the models in Smets and Wouters (2003, 2007) and Christiano, Eichenbaum and Evans (2005): it abstracts from wage rigidities, habit formation in consumption and variable capital utilization.

Households

In our environment, there is a continuum of households indexed by $j \in [0;1]$. Each household maximizes the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(x_t(j)) - Ah_t(j) + \frac{\chi_t}{1-\nu_m} \left[\frac{A}{Z_*^{1/(1-\alpha)}} \frac{m_t(j)}{P_t} \right]^{(1-\nu_m)} \right\}, \tag{9}$$

which is additively separable in consumption $x_t(j)$, labor supply $h_t(j)$ and real money balances $m_t(j)/P_t$. Here β stands for the discount factor, A denotes disutility of labor, ν_m controls the elasticity of money demand and χ_t is an aggregate preference shifter that affects households' marginal utility from holding real money balances.[3] The law of motion for χ_t is:

$$\ln \chi_t = (1-\rho_\chi) \ln \chi_* + \rho_\chi \ln \chi_{t-1} + \varepsilon_{\chi,t}, \qquad \text{where } \varepsilon_{\chi,t} \sim N(0, \sigma_\chi^2) \tag{10}$$

We assume that households are able to trade on a complete set of Arrow-Debreu (A-D) securities, which are contingent on all aggregate and idiosyncratic events $\omega \in \Omega$ in the economy. Let $a_{t+1}(j)(\omega)$ denote the quantity of A-D securities (that pay 1 unit of consumption in period $t+1$ in the event ω) acquired by household j at time t at real price $q_{t+1,t}(j)$. Then household j's budget constraint in nominal terms is given by:

$$\begin{aligned} P_t x_t(j) + P_t i_t(j) + b_{t+1}(j) + m_{t+1}(j) + P_t \int_\Omega q_{t+1,t}(j) a_{t+1}(j)(\omega) d\omega = \\ = P_t W_t h_t(j) + P_t R_t^k k_t(j) + \Pi_t + R_{t-1} b_t(j) + m_t(j) + P_t a_t(j) - T_t \end{aligned} \tag{11}$$

where P_t is the period t price of the final good, $i_t(j)$ is investment, $b_t(j)$ and $m_t(j)$ are government bond and money holdings, R_t is the gross nominal interest rate on government bonds, W_t and R_t^k are the real wage and real return on capital earned by households, Π_t

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[3] As in Aruoba and Schorfheide (2009), scaling $m_t(j)/P_t$ by a factor $A/Z_*^{1/(1-\alpha)}$ can be viewed as re-parameterization of χ_t, in which the steady-state money velocity remains constant when we move around A and Z_*.

stands for profits from owning the firms, and T_t is the nominal amount of lump-sum taxes paid. Households also accumulate capital $k_t(j)$ according to the following law of motion:

$$k_{t+1}(j) = (1-\delta)k_t(j) + \left[1 - S\left(\frac{i_t(j)}{i_{t-1}(j)}\right)\right] i_t(j), \tag{12}$$

where δ is the depreciation rate and $S(\bullet)$ is an adjustment cost function satisfying $S(1)=0$, $S'(1)=0$ and $S''(1)>0$.

The problem of each household j is to maximize the utility function (9) subject to budget constraint (11) and capital accumulation equation (12) for all t. Associate Lagrange multipliers $\lambda_t(j)$ and $Q_t(j)$ with constraints (11) and (12), respectively. The first-order conditions are provided in Appendix A1. We do not take the first-order conditions with respect to A-D securities holdings $a_{t+1}(j)$ explicitly, because we make use of the result in Erceg, Henderson and Levin (2000). This result says that under the assumption of complete markets for A-D securities and under the additive separability of labor and money balances in households' utility, the equilibrium price of A-D securities will be such that optimal consumption will not depend on idiosyncratic shocks. Hence, all households will share the same marginal utility of consumption, and the Lagrange multiplier $\lambda_t(j)$ will also be the same across all households: $\lambda_t(j) = \lambda_t$, all j and t. This implies that in equilibrium all households will choose the same consumption, money and bond holdings, investment and capital. Note that we don't have wage rigidity in this model: therefore, the choice of optimal labor will also be same. Therefore we can safely drop index j from all household-related conditions and variables and proceed accordingly.

Let us define the stochastic discount factor $\Xi^p_{t+1|t}$ that the firms – whose behavior we are going to describe shortly – will use to value streams of future profits:

$$\Xi^p_{t+1|t} = \frac{\lambda_{t+1}}{\lambda_t} = \frac{U'(x_{t+1})}{U'(x_t)} \frac{1}{\pi_{t+1}}, \tag{13}$$

where $\pi_t = P_t / P_{t-1}$ denotes final good price inflation.

Final Good Firms

There is single final good Y_t in our economy manufactured by combining a continuum of intermediate goods $Y_t(i)$ indexed by $i \in [0;1]$ according to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right)^{(1+\lambda)}, \tag{14}$$

where the elasticity of substitution between any goods i and j is $\dfrac{1+\lambda}{\lambda}$.

The final good firms purchase intermediate goods in the market, package them into a composite final good, and sell the final good to households. These firms are perfectly competitive and maximize one-period profits subject to production function (14), taking as given intermediate goods prices $P_t(i)$ and own output price P_t:

$$\begin{aligned} &\max_{Y_t, Y_t(i)} \quad P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \\ &\text{s.t.} \quad Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right)^{(1+\lambda)} \end{aligned} \tag{15}$$

The first-order condition leads to the optimal demand for good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda)}{\lambda}} Y_t. \tag{16}$$

Since final good firms are perfectly competitive and there is free entry, they earn zero profits in equilibrium, which, together with optimal demand (16), yields the price of the final good:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda}} di \right]^{-\lambda}. \tag{17}$$

Intermediate Goods Firms

Our economy is populated by a continuum of intermediate goods firms. Each intermediate goods firm i uses the following technology to produce its output:

$$Y_t(i) = \max\left\{ Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)} - \tilde{F}, 0 \right\}, \tag{18}$$

where $K_t(i)$ is the amount of capital that the firm i rents from households, $H_t(i)$ is the amount of labor input and Z_t is the level of neutral technology evolving according to the law of motion:

$$\ln Z_t = (1-\rho_Z) \ln Z_* + \rho_Z \ln Z_{t-1} + \varepsilon_{Z,t}, \text{ where } \varepsilon_{Z,t} \sim N(0, \sigma_Z^2). \tag{19}$$

Parameter α stands for the capital share of production, while parameter $\tilde{F}$ controls the amount of fixed costs in production that guarantee that the firm's economic profits will be zero in the steady state. Unlike with the final good producers, we do not allow for free entry or exit on the part of the intermediate goods firms.

All intermediate goods producers are *monopolistically competitive*, in that they take all factor prices (W_t and R_t^k), as well as the prices of other firms, as given, but can optimally choose their own price $P_t(i)$ subject to optimal demand (16) for good i from final good firms. Intermediate firms solve a two-stage optimization problem.

In the first stage, the firms hire capital and labor from households to minimize total nominal costs:

$$\begin{aligned} &\min_{K_t(i),H_t(i)} && P_t W_t H_t(i) + P_t R_t^k K_t(i) \\ &\text{s.t.} && Y_t(i) = \max\left\{Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)} - \tilde{F}, 0\right\} \end{aligned} \tag{20}$$

Assuming interior solution, optimality conditions imply ($\eta_t(i)$ is the Lagrange multiplier attached to (18)):

$$\begin{aligned} P_t W_t &= \eta_t(i) P_t(i)(1-\alpha) Z_t K_t(i)^{\alpha} H_t(i)^{-\alpha} \\ P_t R_t^k &= \eta_t(i) P_t(i) \alpha Z_t K_t(i)^{\alpha-1} H_t(i)^{1-\alpha} \end{aligned}$$

Take the ratio of two conditions to obtain:

$$\frac{K_t(i)}{H_t(i)} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} \tag{21}$$

If we define aggregate capital stock $K_t = \int_0^1 K_t(i)di$ and aggregate labor $H_t = \int_0^1 H_t(i)di$, integrating both sides of (21) yields:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} H_t \tag{22}$$

Now we can factorize total real variable cost $VC_t(i)$ into real marginal cost MC_t and the variable part of firm i's output $Y_t^{\text{var}}(i) = Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)}$:

$$VC_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) H_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) \frac{1}{Z_t} \left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha} Y_t^{\text{var}}(i) \tag{23}$$

Plugging in the optimal capital labor ratio (21), real marginal cost MC_t turns out to be the same across all intermediate goods firms:

$$MC_t \overset{def}{=} \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) \frac{1}{Z_t} \left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha} = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)} \frac{W_t^{1-\alpha}\left(R_t^k\right)^{\alpha}}{Z_t} \tag{24}$$

The intuition is that all firms face identical technology shocks and hire inputs at the same factor prices.

In the second stage, all intermediate goods firms have to choose their own price $P_t(i)$ that maximizes total discounted nominal profits subject to demand curve (16). Given optimal choices of inputs from the first stage, the one-period nominal profits of firm i are:

$$\begin{aligned} \Pi_t(i) &= P_t(i)Y_t(i) - P_t W_t \tilde{H}_t(i) - P_t R_t^k \tilde{K}_t(i) = P_t(i)Y_t(i) - P_t\left(MC_t Y_t^{\text{var}}(i)\right) = \\ &= \left(P_t(i) - P_t MC_t\right) Y_t(i) - P_t MC_t \tilde{F} \end{aligned} \tag{25}$$

Note that we can ignore the term $P_t MC_t \tilde{F}$ since it doesn't depend on a firm's choice.

We assume that intermediate goods firms face *nominal price rigidity* a la Calvo (1983). In each period, a fraction $(1-\zeta)$ of firms can optimize their prices. As in Aruoba and Schorfheide (2009), we modify Calvo's original set-up and assume that all other firms cannot adjust their prices and can only index $P_t(i)$ by a geometric weighted average of the fixed rate π_{**} and of the previous period's inflation π_{t-1}, with weights $(1-\iota)$ and ι respectively. The corresponding price adjustment factor is:

$$\pi^{adj}_{t+s|t} = \begin{cases} 1, & s=0 \\ \prod_{l=1}^{s}\left(\pi^{\iota}_{t+l-1}\pi^{(1-\iota)}_{**}\right), & s>0 \end{cases} \tag{26}$$

The firms allowed to re-optimize must choose the optimal price $P^o_t(i)$ that maximizes the discounted value of profits in all states of nature in which the firm faces that price in the future:

$$\begin{aligned} &\max_{P^o_t(i)} \quad \Xi^p_{t|t}(P^o_t(i)-P_tMC_t)Y_t(i)+E_t\left\{\sum_{s=1}^{\infty}(\zeta\beta)^s\,\Xi^p_{t+s|t}(P^o_t(i)\pi^{adj}_{t+s|t}-P_{t+s}MC_{t+s})Y_{t+s}(i)\right\} \\ &\text{s.t.} \quad Y_{t+s}(i)=\left[\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{t+s}, \qquad s=0,1,2,.... \end{aligned} \tag{27}$$

Notice that $\beta^s\Xi^p_{t+s|t}$ is the period t value of a future dollar for the consumer/household in period $t+s$.

Since we consider only a *symmetric equilibrium* in which all firms re-optimizing their prices will choose the same price $P^o_t(i)=P^o_t$, we can drop the indices i from firms' conditions and variables. Given (17) and Calvo pricing, the aggregate price index P_t should evolve as:

$$P_t = \left[(1-\zeta)\left(P^o_t\right)^{-\frac{1}{\lambda}}+\zeta\left(\pi^{\iota}_{t-1}\pi^{(1-\iota)}_{**}P_{t-1}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{28}$$

and, dividing by P_{t-1} and defining $p^o_t = P^o_t / P_t$, yields:

$$\pi_t = \left[(1-\zeta)\left(\pi_t p^o_t\right)^{-\frac{1}{\lambda}}+\zeta\left(\pi^{\iota}_{t-1}\pi^{(1-\iota)}_{**}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{29}$$

As is standard in the literature, the first-order conditions (Appendix A2) of intermediate firms' problem (27) connect the evolution of inflation to the dynamics of real marginal costs and output, and thus imply the New Keynesian Phillips curve.

Monetary and Fiscal Policy

The central bank sets the one-period nominal interest rate on public debt via a Taylor-type interest rate feedback rule responding to deviations of inflation and real output from their target levels:

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R} \left(\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{Y_t}{Y_*}\right)^{\psi_2}\right)^{(1-\rho_R)} e^{\varepsilon_{R,t}}, \qquad \text{where } \varepsilon_{R,t} \sim N(0,\sigma_R^2) \tag{30}$$

where R_*, π_* and Y_* are the steady-state values of the gross nominal interest rate, final good inflation and real final output, respectively. Parameter ρ_R is introduced to control for the degree of interest rate smoothing that we observe in the postwar U.S. data. Also, the central bank supplies enough money balances M_t to meet demand from households, given the desired nominal interest rate.

Every period the government spends G_t in real terms to purchase goods in the final goods market, issues nominal bonds B_{t+1} that pay R_t in gross interest next period and collects nominal lump-sum taxes from households T_t. Each period, the combined government (central bank + Treasury) budget constraint is:

$$P_t G_t + R_{t-1} B_t + M_t = T_t + B_{t+1} + M_{t+1} \tag{31}$$

Real government spending is modeled as a stochastic fraction of total output (i.e., fiscal policy is passive):

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t, \tag{32}$$

where g_t is an exogenous process shifting G_t:

$$\ln g_t = (1-\rho_g)\ln g_* + \rho_g \ln g_{t-1} + \varepsilon_{g,t}, \text{ where } \varepsilon_{g,t} \sim N(0,\sigma_g^2). \tag{33}$$

Aggregation

We now derive the aggregate demand condition. To that end, we integrate budget constraints across all households and combine the result with the government budget constraint (31), introducing aggregate variables – consumption $X_t = \int_0^1 x_t(j)dj$ and investment $I_t = \int_0^1 i_t(j)dj$:

$$P_t X_t + P_t I_t + P_t G_t = P_t W_t H_t + P_t R_t^k K_t + \Pi_t . \tag{34}$$

We derive the expression for aggregate profits Π_t from intermediate firms' problems, combine it with (34) and divide the result by P_t to obtain the *aggregate demand condition*:

$$X_t + I_t + G_t = Y_t \tag{35}$$

From the supply side, the aggregate output of intermediate goods firms $\bar{Y}_t$ is given by:

$$\bar{Y}_t = \int_0^1 Z_t K_t(i)^\alpha H_t(i)^{(1-\alpha)} di - \tilde{F} = Z_t \int_0^1 \left(\frac{K_t(i)}{H_t(i)} \right)^\alpha H_t(i) di - \tilde{F} = Z_t K_t{}^\alpha H_t{}^{(1-\alpha)} - \tilde{F}, \tag{36}$$

where we have used the fact that the capital/labor ratio is constant across firms. However, from (16):

$$\bar{Y}_t = \int_0^1 Y_t(i) di = Y_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda)}{\lambda}} di \tag{37}$$

Hence, the *aggregate supply condition* becomes:

$$Y_t = \frac{1}{D_t} (Z_t K_t^\alpha H_t^{1-\alpha} - \tilde{F}), \tag{38}$$

with $D_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda)}{\lambda}} di$ measuring the extent of aggregate loss of efficiency caused by price dispersion across intermediate goods firms. In Appendix A3, we show that aggregate price dispersion D_t evolves according to:

$$D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t} \right)^\iota \left(\frac{\pi_{**}}{\pi_t} \right)^{(1-\iota)} \right]^{-\frac{(1+\lambda)}{\lambda}} D_{t-1} + (1-\zeta) \left[\frac{P_t^o}{P_t} \right]^{-\frac{(1+\lambda)}{\lambda}} \tag{39}$$

For convenience, we collect all DSGE model parameters in the vector $\boldsymbol{\theta}$ and stack all innovations in vector $\varepsilon_t = [\varepsilon_{Z,t}, \varepsilon_{\chi,t}, \varepsilon_{g,t}, \varepsilon_{R,t}]'$. We then derive a log-linear approximation to the system of equilibrium conditions (summarized in Appendix A4 and A5) around its deterministic steady state. The resulting linear rational expectations system is solved by the method described in Sims (2002).

III. Econometric Methodology

In this section, we first provide the details on a Markov Chain Monte Carlo (MCMC) algorithm to estimate the data-rich DSGE model, including the choice of the prior for factor loadings. Second, we present the novel speed-up suggested by Jungbacker and Koopman (2008), which enhances the speed of our Bayesian estimation procedure.

A. Estimation of the Data-Rich DSGE Model

As discussed in the previous section, the state-space representation of our data-rich DSGE model consists of a transition equation of model states S_t and a set of measurement equations relating the states[4] to data X_t:

$$\underbrace{S_t}_{N\times 1} = \underbrace{\mathbf{G}(\boldsymbol{\theta})}_{N\times N}\underbrace{S_{t-1}}_{N\times 1} + \underbrace{\mathbf{H}(\boldsymbol{\theta})}_{N\times N_\varepsilon}\underbrace{\varepsilon_t}_{N_\varepsilon\times 1} \tag{40}$$

$$\underbrace{X_t}_{J\times 1} = \underbrace{\boldsymbol{\Lambda}(\boldsymbol{\theta})}_{J\times N}\underbrace{S_t}_{N\times 1} + \underbrace{e_t}_{J\times 1} \tag{41}$$

$$e_t = \boldsymbol{\Psi} e_{t-1} + v_t, \tag{42}$$

where $\varepsilon_t \sim iid\, N(\mathbf{0},\mathbf{Q}(\boldsymbol{\theta}))$, $v_t \sim iid\, N(\mathbf{0},\mathbf{R})$ and where $\mathbf{Q}(\boldsymbol{\theta})$, $\mathbf{R}$ and $\boldsymbol{\Psi}$ are assumed diagonal. An essential feature of a data-rich framework is that the panel dimension of data set J is much higher than the number of DSGE model states N. For convenience, collect state-space matrices from the measurement equation into $\Gamma = \{\boldsymbol{\Lambda}(\boldsymbol{\theta}),\boldsymbol{\Psi},\mathbf{R}\}$ and DSGE states-factors into $S^T = \{S_1, S_2, \ldots, S_T\}$. Because of the normality of structural shocks ε_t and measurement error innovations v_t, system (40)-(42) is a linear Gaussian state-space model and the likelihood function of data $p(X^T \mid \boldsymbol{\theta},\Gamma)$ can be evaluated using a Kalman filter.

Following Boivin and Giannoni (2006), we use Bayesian techniques to estimate the unknown model parameters $(\boldsymbol{\theta},\Gamma)$. We combine prior $p(\boldsymbol{\theta},\Gamma) = p(\Gamma \mid \boldsymbol{\theta})p(\boldsymbol{\theta})$ with the likelihood function $p(X^T \mid \boldsymbol{\theta},\Gamma)$ to obtain the posterior distribution of parameters given data:

$$p(\boldsymbol{\theta},\Gamma \mid X^T) = \frac{p(X^T \mid \boldsymbol{\theta},\Gamma)\,p(\boldsymbol{\theta},\Gamma)}{\int p(X^T \mid \boldsymbol{\theta},\Gamma)\,p(\boldsymbol{\theta},\Gamma)\,d\boldsymbol{\theta}\,d\Gamma} \tag{43}$$

We use Markov Chain Monte Carlo (MCMC) method to estimate posterior density $p(\boldsymbol{\theta},\Gamma \mid X^T)$ by constructing a Markov chain with the property that its limiting invariant distribution is our posterior distribution. Similarly to Boivin and Giannoni (2006), the Markov chain is constructed by the Gibbs sampling method with a Metropolis-within-Gibbs step to generate draws from the posterior distribution $p(\boldsymbol{\theta},\Gamma \mid X^T)$ and to compute the approximations to posterior means and covariances of parameters of interest.

But before we turn to describing the Gibbs sampler, we must elaborate on *how we connect the DSGE model states to data indicators*. This is important, because, unlike in Boivin and Giannoni (2006), the link is primarily through the prior on factor loadings $\boldsymbol{\Lambda}(\boldsymbol{\theta})$. The priors for the rest of the parameters ($\boldsymbol{\theta}$, $\boldsymbol{\Psi}$ and $\mathbf{R}$) are discussed in detail in the section "Empirical

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[4] In measurement equations (41) we keep only the non-redundant state variables of a DSGE model. Because some of the DSGE states are merely linear combinations of the other states, one can interpret this as minimum-state-variable approach in the spirit of McCallum (1983, 1999, 2003). Here, though, the main rationale is to avoid multicollinearity on the right hand side of (41). We always set the corresponding factor loadings in $\boldsymbol{\Lambda}$ equal to zero.

Results: Priors" below. Recall that we have *core* data series that measure specific model concepts and *non-core* informational variables that are related to all states of the DSGE model. Consider the following hypothetical example:

$$\begin{matrix} \text{core} \\ \\ \text{non-core} \end{matrix} \left\{ \underbrace{\begin{bmatrix} \text{output \#1} \\ \text{output \#2} \\ \text{inflation \#1} \\ \text{inflation \#2} \\ \vdots \\ \hline \text{exchange rate} \\ X_t^{S,\,rest} \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} \lambda'_{Y1} \\ \lambda'_{Y2} \\ \lambda'_{\pi 1} \\ \lambda'_{\pi 2} \\ \vdots \\ \hline \lambda'_{ER} \\ \mathbf{\Lambda}_S \end{bmatrix}}_{\mathbf{\Lambda}(\boldsymbol{\theta})} \cdot \underbrace{\begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \vdots \end{bmatrix}}_{S_t} + \underbrace{\begin{bmatrix} e_t^F \\ \hline e_t^S \end{bmatrix}}_{e_t} \right. \tag{44}$$

As a matter of general principle, for each of the core series we center the prior mean of λ's at regular-DSGE-model-implied factor loadings of a corresponding model concept. In the example above, this corresponds to the conditional prior for core loadings being:

$$\begin{aligned} p(\lambda_{Y1} \mid \boldsymbol{\theta}) = p(\lambda_{Y2} \mid \boldsymbol{\theta}) = N\left([1,0,0,...,0]', \Omega(\boldsymbol{\theta})\right) \\ p(\lambda_{\pi 1} \mid \boldsymbol{\theta}) = p(\lambda_{\pi 2} \mid \boldsymbol{\theta}) = N\left([0,4,0,...,0]', \Omega(\boldsymbol{\theta})\right). \end{aligned} \tag{45}$$

This means that in regular DSGE model, the output #1 in the data is equal to 1 times output $\hat{Y}_t$ in the model, and inflation #1 in the data is equal to 4 times inflation $\hat{\pi}_t$ in the model (conversion from quarterly to annual inflation). In the data-rich DSGE model, we do not impose $\lambda_{Y,0} = [1,0,0,...,0]'$ and $\lambda_{\pi,0} = [0,4,0,...,0]'$ on loadings λ_Y and λ_π, but instead use them to center the prior means for λ_Y and λ_π. This is different from Boivin and Giannoni (2006), who restrict core factor loadings λ_Y and λ_π to be either $\lambda_{Y,0}$ and $\lambda_{\pi,0}$ or proportional to these.

For non-core series, we center the prior mean of factor loadings at zero vector with an identity covariance matrix. In terms of example (44), the conditional prior is:

$$p(\lambda_{ER} \mid \boldsymbol{\theta}) = p(\mathbf{\Lambda}'_{S,k} \mid \boldsymbol{\theta}) = N\left([0,0,0,...,0]', \mathbf{I}_N\right), \tag{46}$$

where sub-index *k* selects one row from matrix $\mathbf{\Lambda}_S$.

Note that prior means for core loadings may in general depend on DSGE model parameters $\boldsymbol{\theta}$. For instance, if core series contain a measure of inverse money velocity IVM_t, then the DSGE model counterpart $\hat{M}_t - \hat{Y}_t$ (real money balances minus real output in logs) depends on state S_t indirectly, say, via $\hat{M}_t - \hat{Y}_t = d_{IVM}(\boldsymbol{\theta}) S_t$. As a result, the conditional prior for loadings in the IVM measurement equation would be $p(\lambda_{IVM_1} \mid \boldsymbol{\theta}) = N\left(d_{IVM}(\boldsymbol{\theta})', \Omega(\boldsymbol{\theta})\right)$.

Also note that to prevent the data-rich DSGE model from drifting too far away from parameter estimates of a regular DSGE model and to fix the scale of the estimated DSGE model state variables, we make the prior for one of the core series within each core subgroup *perfectly tight*. In example (44), we have two subgroups of core series – output and inflation. This implies, without loss of generality, the perfectly tight prior on loadings in the output #1 and inflation #1 equations. Therefore, we write $\mathbf{\Lambda(\theta)}$ to underscore that some loadings will explicitly depend on the DSGE model's structural parameters.

Now let us turn to the description of our Gibbs sampler. MCMC implementation for the linear Gaussian state-space model (40)-(42) is based on the following conditional posterior distributions:

$$p(\Gamma \mid \boldsymbol{\theta}; X^T) \quad p(S^T \mid \Gamma, \boldsymbol{\theta}; X^T) \quad p(\Gamma \mid S^T, \boldsymbol{\theta}; X^T) \quad p(\boldsymbol{\theta} \mid \Gamma; X^T) \tag{47}$$

Essentially, the Gibbs sampler iterates on conditional posterior densities $p(\Gamma \mid \boldsymbol{\theta}; X^T)$ and $p(\boldsymbol{\theta} \mid \Gamma; X^T)$ to generate draws from the joint posterior distribution $p(\boldsymbol{\theta}, \Gamma \mid X^T)$ of the state-space parameters Γ and the structural DSGE model parameters $\boldsymbol{\theta}$. It uses an intermediate step to draw DSGE states S^T, because this simplifies sampling the elements of Γ conditional on S^T and $\boldsymbol{\theta}$. The sampling of $\boldsymbol{\theta}$ relies on a Metropolis-within-Gibbs step, since the conditional posterior density $p(\boldsymbol{\theta} \mid \Gamma; X^T)$ is generally intractable.

The main steps of the *Gibbs sampler* are (we provide full details in Appendix B):

1. Specify initial values $\boldsymbol{\theta}^{(0)}$ and $\Gamma^{(0)}$.
2. Repeat for $g = 1, 2, ..., n_{sim}$

   2.1. Solve the DSGE model numerically at $\boldsymbol{\theta}^{(g-1)}$ and obtain matrices $\mathbf{G}(\boldsymbol{\theta}^{(g-1)})$, $\mathbf{H}(\boldsymbol{\theta}^{(g-1)})$ and $\mathbf{Q}(\boldsymbol{\theta}^{(g-1)})$

   2.2. Draw from $p(\Gamma \mid \boldsymbol{\theta}^{(g-1)}; X^T)$:

   a) Generate unobserved states $S^{T,(g)}$ from $p(S^T \mid \Gamma^{(g-1)}, \boldsymbol{\theta}^{(g-1)}; X^T)$ using the Carter-Kohn (1994) forward-backward algorithm;

   b) Generate state-space parameters $\Gamma^{(g)}$ from $p(\Gamma \mid S^{T,(g)}, \boldsymbol{\theta}^{(g-1)}; X^T)$ by drawing from a complete set of known conditional densities $[\mathbf{R} \mid \mathbf{\Lambda}, \mathbf{\Psi}; \Xi]$, $[\mathbf{\Lambda} \mid \mathbf{R}, \mathbf{\Psi}; \Xi]$ and $[\mathbf{\Psi} \mid \mathbf{\Lambda}, \mathbf{R}; \Xi]$, where $\Xi = \left\{S^{T,(g)}, \boldsymbol{\theta}^{(g-1)}, X^T\right\}$.

   2.3. Draw DSGE parameters $\boldsymbol{\theta}^{(g)}$ from $p(\boldsymbol{\theta} \mid \Gamma^{(g)}; X^T)$ using Metropolis step:

   a) Propose

$$\boldsymbol{\theta}^* \sim q(\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(g-1)}; \Gamma^{(g)}) \tag{48}$$

   b) Draw $u \sim Uniform(0,1)$ and set

$$\boldsymbol{\theta}^{(g)} = \begin{cases} \boldsymbol{\theta}^* & \text{if } u \le \alpha(\boldsymbol{\theta}^* \mid \Gamma^{(g)}, \boldsymbol{\theta}^{(g-1)}) \\ \boldsymbol{\theta}^{(g-1)} & \text{otherwise} \end{cases} \tag{49}$$

where acceptance probability $\alpha(\bullet)=\min\left\{1, r(\boldsymbol{\theta}^{(g-1)},\boldsymbol{\theta}^{*},\Gamma^{(g)})\right\}$ and

$$r(\boldsymbol{\theta}^{(g-1)},\boldsymbol{\theta}^{*},\Gamma^{(g)})=\frac{p(\boldsymbol{\theta}^{*},\Gamma^{(g)}\mid X^{T})}{p(\boldsymbol{\theta}^{(g-1)},\Gamma^{(g)}\mid X^{T})}=\frac{p(X^{T}\mid\boldsymbol{\theta}^{*},\Gamma^{(g)})\,p(\Gamma^{(g)}\mid\boldsymbol{\theta}^{*})\,p(\boldsymbol{\theta}^{*})}{p(X^{T}\mid\boldsymbol{\theta}^{(g-1)},\Gamma^{(g)})\,p(\Gamma^{(g)}\mid\boldsymbol{\theta}^{(g-1)})\,p(\boldsymbol{\theta}^{(g-1)})}. \qquad (50)$$

3. Return $\left\{\boldsymbol{\theta}^{(g)},\Gamma^{(g)}\right\}_{g=1}^{n_{sim}}$

The Carter-Kohn (1994) algorithm in step 2.2.(a) proceeds as follows. First, it applies a Kalman filter to the state-space system (40)-(42) to generate filtered DSGE states $\hat{S}_{t|t}$, $t=1..T$. Then, starting from $\hat{S}_{T|T}$, it rolls back in time along Kalman smoother recursions to draw elements of $S^{T,(g)}$ from a sequence of conditional Gaussian distributions.

The intermediate step to generate DSGE model states $S^{T,(g)}$ is used to facilitate sampling state-space matrices $\Gamma^{(g)}$ in 2.2.(b). Conditional on $S^{T,(g)}$, the elements of matrices $\Gamma^{(g)}=\left\{\boldsymbol{\Lambda}^{(g)},\boldsymbol{\Psi}^{(g)},\mathbf{R}^{(g)}\right\}$ are the parameters of simple linear regressions (41)-(42) and we can draw them equation by equation using the approach of Chib and Greenberg (1994). It is a straightforward procedure, since we assume conjugate priors for Γ and conditional posterior densities are all of known functional forms.

To generate DSGE model parameters $\boldsymbol{\theta}^{(g)}$, we introduce Metropolis step 2.3. It is required because density $p(\boldsymbol{\theta}\mid\Gamma;X^{T})$ is generally intractable and cannot be easily factorized into known conditionals. We choose to use the *random-walk version of Metropolis step* (e.g., An and Schorfheide, 2007) in which the proposal density $q(\boldsymbol{\theta}'\mid\boldsymbol{\theta})$ is a multivariate Student-t with mean equal to the previous draw $\boldsymbol{\theta}^{(g-1)}$ and a covariance matrix proportional to the inverse Hessian from the *regular* DSGE model[5] evaluated at the posterior mode.

To *initialize* our Gibbs sampler, we first run a regular DSGE model estimation (see footnote 5), compute the posterior mean of DSGE model parameters and generate smoothed model states $S^{T,reg}$. Then we take the rich panel of macro and financial series X^{T} and run equation-by-equation OLS regressions of X_{k}^{T} on smoothed DSGE states $S^{T,reg}$ to back out initial values for $\boldsymbol{\Lambda}$, $\boldsymbol{\Psi}$ and $\mathbf{R}$.

Under regularity conditions satisfied here for the linear Gaussian state-space model, the Markov chain $\{\boldsymbol{\theta}^{(g)},\Gamma^{(g)}\}$ constructed by the Gibbs sampler above converges to its invariant distribution and, starting from some $g>\bar{g}$, contains draws from the posterior distribution of interest $p(\boldsymbol{\theta},\Gamma\mid X^{T})$. Sample averages of these draws (or their appropriate transformations)

[5] Running a bit ahead, in our empirical analysis this regular DSGE estimation features the same underlying theoretical DSGE model as in the data-rich version, but only four (equal to the number of shocks) core observables assumed to have been measured without errors. These core observables are (appropriately transformed) real GDP, GDP deflator inflation, the federal funds rate and the inverse velocity of money based on M2S. See details in the Data and Transformations section. Also see the notes to Table D3.

converge almost surely to respective population moments under our posterior density (Tierney 1994, Chib 2001, Geweke 2005).

B. Speed-Up: Jungbacker and Koopman 2008

The data-rich DSGE model (40)-(42) is potentially a high-dimensional object (the panel dimension *J* could be as high as 100+), and therefore, the MCMC algorithm outlined above spends a lot of time evaluating the likelihood function with the Kalman filter and sampling the DSGE states S_t at every iteration. To reduce the computational costs associated with a likelihood-based analysis of dynamic factor models (of which our data-rich DSGE model is a special case), Jungbacker and Koopman (2008) proposed to use the Kalman filter and smoother techniques based on a *lower-dimensional transformation* of the original data vector X_t.

Without loss of generality, consider the generic data-rich DSGE model introduced in Section II. The first-order dynamics of errors e_t allow us to rewrite the system (2), (7)-(8) in state-space form as follows:

$$\tilde{X}_t = \underbrace{\left[\boldsymbol{\Lambda}(\boldsymbol{\theta}) \;\vdots\; -\boldsymbol{\Psi}\boldsymbol{\Lambda}(\boldsymbol{\theta})\right]}_{\tilde{\boldsymbol{\Lambda}}} \underbrace{\begin{bmatrix} S_t \\ S_{t-1} \end{bmatrix}}_{\tilde{F}_t} + v_t \tag{51}$$

$$\tilde{F}_t = \underbrace{\begin{bmatrix} \mathbf{G}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{G}}} \tilde{F}_{t-1} + \underbrace{\begin{bmatrix} \mathbf{H}(\boldsymbol{\theta}) \\ \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{H}}} \varepsilon_t, \tag{52}$$

where we denoted $\tilde{X}_t = X_t - \boldsymbol{\Psi} X_{t-1}$. Collect all the matrices in $\boldsymbol{\Theta} = \left\{\boldsymbol{\Lambda}, \boldsymbol{\Psi}, \mathbf{R}, \tilde{\mathbf{G}}, \tilde{\mathbf{H}}, \mathbf{Q}\right\}$. Suppose that the proposed lower-dimensional transformation of data vector $\tilde{X}_t$ is implemented by some $J \times J$ invertible matrix $\mathbf{A}$ such that $\tilde{X}_t^* = \mathbf{A}\tilde{X}_t$, $t = 1..T$. Also, suppose that we partition $\tilde{X}_t^*$ and $\mathbf{A}$ as below:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^L \\ \mathbf{A}^H \end{bmatrix}, \qquad \tilde{X}_t^* = \begin{bmatrix} \tilde{X}_t^L \\ \tilde{X}_t^H \end{bmatrix}, \qquad \text{where } \tilde{X}_t^L = \mathbf{A}^L \tilde{X}_t, \;\; \tilde{X}_t^H = \mathbf{A}^H \tilde{X}_t, \tag{53}$$

with matrices $\mathbf{A}^L$ and $\mathbf{A}^H$ being $m \times J$ and $(J-m) \times J$, $m < J$.

Jungbacker and Koopman (2008) are able to show (Lemma 1, Lemma 2) that you can find a suitable matrix $\mathbf{A}$ such that $\tilde{X}_t^L$ and $\tilde{X}_t^H$ are uncorrelated and only the low-dimensional sub-vector $\tilde{X}_t^L$ depends on DSGE states $\tilde{F}_t$:

$$\begin{aligned} \tilde{X}_t^L &= \mathbf{A}^L \tilde{\boldsymbol{\Lambda}} \tilde{F}_t + v_t^L, \\ \tilde{X}_t^H &= v_t^H, \end{aligned} \qquad \begin{bmatrix} v_t^L \\ v_t^H \end{bmatrix} \sim iidN\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_L & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_H \end{bmatrix} \right), \tag{54}$$

where $\mathbf{\Sigma}_L = \mathbf{A}^L \mathbf{R} \mathbf{A}^{L\prime}$ and $\mathbf{\Sigma}_H = \mathbf{A}^H \mathbf{R} \mathbf{A}^{H\prime}$. Moreover, they show that the knowledge of a high-dimensional matrix $\mathbf{A}^H$ and a data vector $\tilde{X}_t^H$ is not required to estimate the DSGE states $\tilde{F}_t$ and to compute the likelihood of the original model.

In terms of matrix $\mathbf{A}^L$, Jungbacker and Koopman prove that it should be of the form:

$$\mathbf{A}^L = \mathbf{C}\bar{\mathbf{\Lambda}}'\mathbf{R}^{-1}, \quad (55)$$

for some invertible $m \times m$ matrix $\mathbf{C}$ and $J \times m$ matrix $\bar{\mathbf{\Lambda}}$, columns of which form a basis of the column space of $\tilde{\mathbf{\Lambda}}$. In practice, they recommend setting $\bar{\mathbf{\Lambda}} = \tilde{\mathbf{\Lambda}}$ and $\mathbf{C} = \left(\tilde{\mathbf{\Lambda}}'\mathbf{R}^{-1}\tilde{\mathbf{\Lambda}}\right)^{-1}$ in case the matrix of factor loadings $\tilde{\mathbf{\Lambda}}$ has full column rank.

Now that we know $\mathbf{A}^L$ we can sample states $\tilde{F}_t$ using the Carter-Kohn (1994) forward-backward algorithm applied to a lower-dimensional model

$$\tilde{X}_t^L = \mathbf{A}^L \tilde{\mathbf{\Lambda}} \tilde{F}_t + v_t^L, \qquad v_t^L \sim iid\ N(\mathbf{0}, \mathbf{\Sigma}_L) \quad (56)$$

$$\tilde{F}_t = \tilde{\mathbf{G}}\tilde{F}_{t-1} + \tilde{\mathbf{H}}\varepsilon_t, \qquad \varepsilon_t \sim iid\ N(\mathbf{0}, \mathbf{Q}(\mathbf{\theta})) \ .(57)$$

We can also compute the log-likelihood of data $L(\tilde{X} \mid \mathbf{\Theta})$ as

$$L(\tilde{X} \mid \mathbf{\Theta}) = c + L(\tilde{X}^L \mid \mathbf{\Theta}) - \frac{T}{2}\log\frac{|\mathbf{R}|}{|\mathbf{\Sigma}_L|} - \frac{1}{2}\sum_{t=1}^{T}\hat{v}_t'\mathbf{R}^{-1}\hat{v}_t, \quad (58)$$

where $c = -\frac{1}{2}(J-m)T\log(2\pi)$ and $\hat{v}_t = \tilde{X}_t - \left[\bar{\mathbf{\Lambda}}\left(\bar{\mathbf{\Lambda}}'\mathbf{R}^{-1}\bar{\mathbf{\Lambda}}\right)\bar{\mathbf{\Lambda}}'\mathbf{R}^{-1}\right]\tilde{X}_t$. The term $L(\tilde{X}^L \mid \mathbf{\Theta})$ is the log-likelihood of the transformed data evaluated by using a Kalman filter during the forward pass of the Carter-Kohn algorithm on the low-dimensional model (56)-(57).

In the ensuing empirical analysis of a data-rich DSGE model, we have applied the Jungbacker-Koopman algorithm presented in this section to improve the speed of computations. To get a sense of CPU time gains, we have also estimated the model – though on fewer draws – without the speed-up and have found that the "improved" estimation of the data-rich DSGE model runs 2.5 times faster. The CPU gains reported by Jungbacker and Koopman (2008) for a dynamic factor model of a size similar to our data-rich DSGE model are about 11 times faster. Differences in time savings are due to the significant chunk of time that it takes to solve numerically the underlying DSGE model in the data-rich DSGE model estimation, a step absent in the DFM estimation and not affected by the Jungbacker-Koopman speed-up.

IV. DATA AND TRANSFORMATIONS

To estimate the data-rich DSGE model, we employ a large panel of U.S. quarterly macroeconomic and financial time series compiled by Stock and Watson (2008).[6] The panel

[6] The data set is available online at:
http://www.princeton.edu/~mwatson/ddisk/hendryfestschrift_replicationfiles_April28_2008.zip

covers 1959:Q1 – 2006:Q4, however, our sample in this paper spans only 1984:Q1 – 2005:Q4. We focus on this later period primarily for two reasons: (i) to avoid dealing with the issue of the Great Moderation[7]; and (ii) to concentrate on a period with a relatively stable monetary policy regime.

Our data set consists of 12 *core series* that measure specific DSGE model concepts and 77 *non-core* informational series that load on all DSGE states and may contain useful information about the aggregate state of the economy. The core series include three measures of real output (real GDP, the index of total industrial production and the index of industrial production: manufacturing), three measures of price inflation (GDP deflator inflation, personal consumption expenditure (PCE) deflator inflation, and CPI inflation), three indicators of the nominal interest rates (the federal funds rate, the 3-month T-bill rate and the yield on AAA-rated corporate bonds), and three series measuring the inverse velocity of money (IVM based on the M1 aggregate and the M2 aggregate and IVM based on the adjusted monetary base). The 77 non-core series include the measures of real activity, labor market variables, housing indicators, prices and wages, financial variables (interest rate spreads, exchange rate depreciations, credit stocks, stock returns) and, together with appropriate transformations to eliminate trends, are described in Appendix C.

Most of the core series are computed based on the raw indicators from Stock and Watson (2008) database and from the Fred-II database[8] maintained by the Federal Reserve Bank of St. Louis (database mnemonics are in italics). To obtain three measures of *real per-capita output*, we take real GDP (*SW2008::GDP251*), total industrial production (*SW2008::IPS10*) and industrial production in the manufacturing sector (*SW2008::IPS43*), and divide each series by the civilian non-institutional population (*Fred-II::CNP16OV*). We then take the natural logarithm and extract the linear trend by an OLS regression. The resulting detrended series are multiplied by 100 to convert them to percentage deviations from respective means. The *inflation* measures are computed as the first difference of the natural logarithm of the GDP deflator (*SW2008::GDP272A*), of the PCE deflator (*SW2008::GDP273A*), and of the Consumer Price Index – All Items (*SW2008::CPIAUCSL*), all multiplied by 400 to get to the annualized percentages. Our indicators of the *nominal interest rate* are (i) the effective federal funds rate (*SW2008::FYFF*), (ii) the 3-month U.S. Treasury bill rate in the secondary market (*SW2008::FYGM3*) and (iii) the yield on Moody's AAA-rated corporate bonds (*SW2008::FYAAAC*). We use a simple 3-month average to obtain quarterly annualized interest rates from monthly raw data.

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[7] The "Great Moderation" refers to a decline in the volatility of output and inflation observed in the U.S. since the mid-1980s until the recent financial crisis. For evidence and implications, please see Bernanke (2004), Stock and Watson (2002), Kim and Nelson (1999a), and McConnell and Perez-Quiros (2000). The last two papers argue that a break in the volatility of U.S. GDP growth occurred in 1984:Q1.

[8] The Fred-II database is available online at: http://research.stlouisfed.org/fred2/

To generate the appropriate *inverse money velocities*, we take three monetary aggregates: the sweep-adjusted money stock M1 (*CDJ::M1S*), the sweep-adjusted money stock M2 (*CDJ::M2S*) and the monetary base adjusted for changes in reserve requirements (*SW2008::FMFBA*). The sweep-adjusted stocks M1S and M2S are provided by Cynamon, Dutkowsky and Jones (2006)[9] and correct the distortionary impact (on the conventional measures M1 and M2) of the financial innovation that started in the early 1990s. These distortions take the form of underreporting of actual transactions balances and arise because of retail sweep programs and commercial demand deposit sweep programs, in which U.S. banks move a portion of funds from their customer demand deposits or other checkable deposits into instruments with zero reserve requirements. Since our DSGE model does not have any explicit open- economy context, we further adjust the monetary base FMFBA by deducting the amount of U.S. dollar currency held physically outside the United States.[10] We take M1S, M2S and the adjusted FMFBA, divide each series by the nominal GDP (*Fred-II::GDP*) to obtain the respective inverse velocities of money. For each IVM, we take the natural logarithm of the M/GDP ratio and scale it by 100. Finally, we remove the linear deterministic trend from the IVM based on M1S.

Because measurement equations (41) are modeled without intercepts, we estimate the data-rich DSGE model on a demeaned data set. Also, in line with standard practice in the factor literature, we standardize each time series so that its sample variance is equal to unity (however, we do not scale the core series when estimating the data-rich DSGE model).

V. EMPIRICAL RESULTS

In this section, we conduct the empirical analysis of the regular and the data-rich DSGE model. We begin by discussing the choice of the prior distributions of model parameters and then describe the posterior estimates of deep structural parameters in both models. Second, we compare the estimated DSGE state variables from our data-rich and from the regular DSGE model. Finally, we explore the differences that the two models imply about the sources of business cycle fluctuations and about the propagation of structural innovations, notably the monetary policy and technology shocks, to the measures of real output, inflation, interest rates and the real money balances.

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[9] Sweep-adjusted money stocks are available online at: http://www.sweepmeasures.com.

[10] Federal Reserve Board: Flow of Funds Accounts of the United States: Z.1 Statistical Release for March 12, 2009 (available at http://www.federalreserve.gov/releases/z1/20090312/). Table L.204 “Checkable Deposits and Currency”, line 23 (Rest of the world: Currency), unique identifier: Z1/Z1/FL263025003.Q

A. Priors

Since we estimate the regular DSGE model (130) and the data-rich DSGE model (40)-(42) using Bayesian techniques, we have to provide prior distributions for both models' parameters.

In our data-rich DSGE model, we have two groups of parameters: state-space model parameters comprising matrices $\mathbf{\Lambda}$, $\mathbf{\Psi}$ and $\mathbf{R}$, and deep structural parameters $\boldsymbol{\theta}$ of an underlying DSGE model. The prior for the state-space matrices is elicited differently for the core and the non-core data indicators contained in X_t. Let $\mathbf{\Lambda}_k$ and R_{kk} be the factor loadings and a variance of the measurement error innovation for the k^{th} measurement equation, $k = 1..J$.

Regarding the non-core measurement equations, the prior for $\left(\mathbf{\Lambda}_k, R_{kk}\right)$ and for Ψ_{kk} is defined as follows. Similarly to Boivin and Giannoni (2006) and Kose, Otrok and Whiteman (2008), we assume a joint Normal-InverseGamma prior distribution for $\left(\mathbf{\Lambda}_k, R_{kk}\right)$ so that $R_{kk} \sim IG_2(s_0, \nu_0)$ with location parameter $s_0 = 0.001$ and degrees of freedom $\nu_0 = 3$, and the prior mean of factor loadings is centered around the vector of zeros $\mathbf{\Lambda}_k \mid R_{kk} \sim N(\mathbf{\Lambda}_{k,0}, R_{kk}\mathbf{M}_0^{-1})$ with $\mathbf{\Lambda}_{k,0} = \mathbf{0}$ and $\mathbf{M}_0 = \mathbf{I}_N$. The prior for the k^{th} measurement equation's autocorrelation Ψ_{kk}, all k, is $N(0,1)$. We are making it perfectly tight, however, because there could be data series with stochastic trends we seek to capture with potentially highly persistent DSGE states-factors and not with highly persistent measurement errors. This implies that all measurement errors are *iid* mean-zero normal random variables.

In contrast, the prior distribution for the factor loadings in the core measurement equations follows the scheme explained in example (44). Instead of hypothetical "output" and "inflation" groups, we substitute four categories of the core series: real output, inflation, the nominal interest rate, and the inverse velocity of money, with three specific measures within each category, as described in the Data and Transformations section. The joint prior distribution is still Normal-Inverse-Gamma $(\mathbf{\Lambda}_{k,0}, \mathbf{M}_0, s_0, \nu_o)$, but now, for each of the core series, the prior mean of the factor loadings $\mathbf{\Lambda}_{k,0}$ is centered at the regular-DSGE-model-implied factor loadings of a corresponding DSGE model variable (real output $\hat{Y}_t$, inflation $\hat{\pi}_t$, the nominal interest rate $\hat{R}_t$ or the inverse money velocity $\hat{\hat{M}}_t - \hat{Y}_t$), evaluated at the current draw of deep structural parameters $\boldsymbol{\theta}$. The covariance scaling matrix $\mathbf{M}_0$ is assumed diagonal $\mathbf{M}_0 = diag(\mathbf{\Omega}(\boldsymbol{\theta}))$, where $\mathbf{\Omega}(\boldsymbol{\theta})$ is the unconditional covariance matrix of the DSGE model state variables evaluated at a current draw of $\boldsymbol{\theta}$. $\mathbf{M}_0$ is the same across all core measurement equations. This choice implies that the prior will be tighter for the loadings on more volatile DSGE states. A similar approach is pursued in Schorfheide, Sill and Kryshko (2010). The scale s_0 and degrees of freedom ν_0 are the same as for the parameters in the non-core measurement equations above. Finally, as argued in Section III.A, we use a degenerate prior for real GDP, GDP deflator inflation, the federal funds rate and the IVM based on the M2S monetary aggregate.

Our choice of prior distribution for the deep structural parameters of a DSGE model broadly follows Aruoba and Schorfheide (2009). We keep the same prior for the regular and for the data-rich DSGE models that we estimate below. A subset of these parameters that are *fixed* in estimation is reported in Table D1. We choose to have a logarithmic utility of household consumption by fixing $\gamma = 1$. We set the depreciation rate of capital δ to 0.014, which is the average quarterly ratio of the depreciation of fixed assets to the stock of these fixed assets in 1959-2005 (NIPA-FAT11 for stocks, NIPA-FAT13 for depreciation of fixed assets and consumer durables). The steady-state annualized inflation rate π_A is fixed at 2.5 percent – the average GDP deflator inflation in our sample. We implicitly impose the Fischer equation and let the steady-state annualized real interest rate r_A be equal to 2.84 percent. This value is obtained as the average federal funds interest rate in our sample minus π_A. Households' discount factor is therefore $\beta = 1/(1 + r_A/400)$.

We also introduce several normalizations. We normalize to 1 the steady-state real output Y_* and steady-state money demand shock χ_*. We use the average log inverse velocity of money (log[M2S/GDP]) in our sample to pin down $\log(\bar{M}_*/Y_*)$. Finally, as in Aruoba and Schorfheide (2009), we fix $\log(H_*/Y_*)$ to -3.5. This number is derived from the average inverse labor productivity in the data. In our sample, on average a worker produces roughly \$33 of real GDP per hour. Hence, average H/Y in the data is $1/33$. From the average share of government spending (consumption plus investment) in nominal GDP, we calibrate g_* to be 1.2.

We also want our data-rich DSGE model to be broadly consistent – in terms of the conduct of monetary policy – with the other regular DSGE models estimated on post-1983 data. Therefore, we shut down "data-richness" for a moment and estimate our DSGE model on just three standard observables: real GDP, GDP deflator inflation and the federal funds rate. The resulting estimates of the Taylor (1993) rule coefficients were: $\psi_1 = 1.82$, $\psi_2 = 0.18$ and $\rho_R = 0.78$. In the estimation of the data-rich DSGE model, we set the policy rule coefficients to these values. This procedure is similar in spirit to Boivin and Giannoni (2006), who assume that the policy rate R_t is measured in the data by the federal funds rate without an error. This assumption guarantees that the estimated monetary policy rule coefficients will not drift far away from the conventional post-1983 values documented in the literature.

Despite detrending performed on all three measures of real per capita output, they are still highly persistent. To strike a balance between the observed output persistence and the need to have stationarity in the model, we fix the autocorrelation of the technology shock ρ_Z at 0.98. In the intermediate goods-producing sector, we further assume no fixed costs ($\tilde{F} = 0$) and the absence of static indexation for non-optimizing firms ($\pi_{**} = 1$).

The prior distributions for other parameters are summarized in Table D2. The prior for the steady-state related parameters represents the view that the capital share of α in a Cobb-Douglas production function of intermediate goods firms is about 0.3 and that the average markup these firms charge is about 15 percent. The prior for the Calvo (1983) probability ζ

controlling nominal price rigidity is quite agnostic and spans the range of values consistent with fairly rigid and fairly flexible prices. As in Del Negro and Schorfheide (2008), the prior density for the price indexation parameter ι is close to uniform on a unit interval. Parameter ν_m controlling the interest-rate elasticity of money demand is a priori distributed according to a Gamma distribution with mean 20 and standard deviation 5. The existing literature (e.g., Aruoba, Schorfheide 2009, Levin, Onatsky, Williams and Williams 2005, and Christiano, Eichenbaum and Evans 2005) documents fairly large estimates of the money demand elasticity ranging from 10 to 25. The 90 percent interval for the investment adjustment cost parameter S'' spans values that Christiano, Eichenbaum, Evans (2005) find when matching DSGE and vector autoregression impulse response functions. The priors for the parameters determining the exogenous shock processes are taken from Aruoba and Schorfheide (2009). They reflect the belief that the money demand and government spending shocks are quite persistent.

B. Posteriors: Regular vs. Data-Rich DSGE Model

Using the Gibbs sampler with the Metropolis step outlined in Section III.A, we estimate the data-rich DSGE model. In addition, we have also estimated the regular DSGE model using standard Bayesian techniques (Random Walk Metropolis-Hastings algorithm, see An and Schorfheide, 2007). The underlying theoretical New Keynesian core is the same as in the data-rich DSGE model. The difference comes in the measurement equation (41): we keep only four core observable data series (real GDP, GDP deflator inflation, the federal funds interest rate and the inverse velocity of money based on the M2S aggregate), impose the factor loadings as in (130) and assume perfect measurement of all four model concepts (see the notes to Table D3, p.51).

The only parameters of direct interest here are the deep structural parameters $\boldsymbol{\theta}$ of an underlying DSGE model, and we report the posterior means and 90 percent credible intervals of these in the columns of Table D3. We find the capital share of output and the average price markup to be in line with estimates from regular – few observables, perfect measurement – DSGE estimation. We find little evidence of dynamic indexation by intermediate goods firms in both versions of the model. The implied average duration of nominal price contracts is about $1/(1-0.797)$ = 4.9 quarters. On the one hand, this is close to what Aruoba and Schorfheide (2009) find in their money-in-the-utility specification of a DSGE model and what Del Negro and Schorfheide (2008) document under the “standard” agnostic prior about nominal price rigidities (their Table 6, p. 1206). On the other hand, this is much higher than the price contracts duration of about 3 quarters found by Smets and Wouters (2007) and Schorfheide, Sill and Kryshko (2010). In the context of a data-rich DSGE model similar to ours, Boivin and Giannoni’s (2006) estimates imply that the firms change prices very slowly – on average once per at least 7 quarters. The 4.9 quarters found in the data-rich version is quite higher than the duration of price contracts documented for the regular DSGE model

$(1/(1-0.759)=4.15$ quarters). The implication of this difference is that the implied slope of the New Keynesian Phillips curve[11] measuring the elasticity of current inflation to real marginal costs (and to real output) falls from 0.0745 to 0.0517 as we move from the perfect measurement, few observables to a richer data set in estimation of the same underlying DSGE model. This means, for example, that the cost of disinflation associated with achieving a 1 percent reduction in the rate of inflation at the expense of tolerating negative real output growth, as predicted by the data-rich DSGE model, turns out to be more sizable than the output cost of disinflation predicted by the traditional regular DSGE model.

As anticipated, we have obtained a fairly high elasticity of money demand. Our estimate of ν_m in the data-rich DSGE model case implies that a 100-basis-points increase in the interest rate leads to a 3.2 percent decline in real money balances. A very large estimate of the investment adjustment cost parameter (30.8 in data-rich versus 11.1 in the regular DSGE model), as Aruoba and Schorfheide (2009) argue, has something to do with the need to reduce the volatility of the return to capital and to dampen its effect on marginal costs, which in turn affect current inflation through the New Keynesian Phillips curve relationship. This is reasonable given that in our data-rich DSGE model, the industrial production measures of real output are more volatile than the GDP-based measure, while the volatilities of inflation measures are fairly similar. In both models, the money demand shock χ_t turns out to be highly serially correlated, and the persistence of the government spending shock g_t is high as well, but more moderate. In the data-rich environment, this is hardly surprising, since these shocks are now the common factors for a large sub-panel of non-core informational series, many of which are fairly persistent.

C. Estimated States: Regular vs. Data-Rich DSGE Model

Our empirical analysis proceeds by plotting the estimated DSGE state variables from our data-rich DSGE model and from the regular DSGE model.

Figure D1 depicts the posterior means and 90 percent credible intervals of the estimated data-rich DSGE model states. These include three endogenous variables (model inflation $\hat{\pi}_t$, the nominal interest rate $\hat{R}_t$ and real household consumption $\hat{X}_t$) and three structural AR(1)

[11] We say *implied* slope because our underlying theoretical DSGE model is linearized around positive steady-state inflation rate ($\pi_A = 2.5\%$) and assumes the absence of static price indexation by the non-optimizing intermediate goods firms ($\pi_{**} = 1$). This implies that we have a dynamic New Keynesian Phillips curve with additional lags of real marginal costs $\widehat{MC}_t$. In a more conventional model where the non-optimizing intermediate goods firms index their prices to the steady-state inflation rate ($\pi_{**} = \pi_* = 1 + \pi_A/400$), the NK Phillps curve features only current marginal costs, the coefficient next to which γ_{mc} we report:

$$\hat{\pi}_t = \gamma_{\pi 1}\hat{\pi}_{t-1} + \gamma_{\pi 2}E_t(\hat{\pi}_{t+1}) + \gamma_{mc}\widehat{MC}_t$$

where $\gamma_{\pi 1} = \iota/(1+\beta\iota)$, $\gamma_{\pi 2} = \beta/(1+\beta\iota)$ and $\gamma_{mc} = (1-\zeta)(1-\zeta\beta)/(\zeta(1+\beta\iota))$.

shocks (government spending g_t, money demand χ_t and neutral technology Z_t). It is these states that are included in measurement equation (41) with potentially non-zero loadings. The figure depicts as well the smoothed versions of these same variables in a regular DSGE model estimation derived by Kalman smoother at posterior mean of the deep structural parameters.

Four observations stand out. First, all three structural disturbances exhibit large swings and prolonged deviations from zero capturing the persistent low-frequency movements in the data. Second, the estimated data-rich DSGE model states are much *smoother* than their counterparts in the regular DSGE model. The intuition is straightforward. In the data-rich context, the model states are the common components of a large panel of data, and they have to capture well not only a few core macro series (as is the case in the regular DSGE model), but also very many non-core informational series.

The third observation is that the money demand shock χ_t appears to be very different in the data-rich versus the regular DSGE model estimation. The underlying reason is that in the case of the regular DSGE model, it was mainly responsible for capturing the dynamics of the inverse money velocity based on M2S in the small 4-series data set. Once we allow for the rich panel of macro and financial observables, χ_t helps explain other series as well (for example, housing variables and non-GDP measures of real output – see Table D4), yet at the cost of the fit for the IVM_M2S. The fourth observation is a counterfactual behavior of government spending shock g_t and real consumption $\hat{X}_t$ during recessions: the former tends to fall and the latter to rise when times are bad. In reality, of course, it is the other way around: as a recession unfolds, real consumption falls and government purchases are usually intensified to mitigate the negative impact of the recession on aggregate demand. The estimated path of g_t would make sense, however, if we think of it as a general aggregate demand shock not specifically connected to government purchases. In spite of our DSGE model being able to track well the total output dynamics, it cannot properly discriminate the components, in particular $\hat{X}_t$. The solution would seem to be to enlarge the model by incorporating, say, an investment-specific technology shock a la Greenwood, Hercowitz and Krusell (1998) and to make the real consumption in the data one of the core observables, as for example is done in Smets and Wouters (2007) and Boivin and Giannoni (2006).

D. Sources of Business Cycle Fluctuations

Another dimension along which the data-rich DSGE model and the regular (few observables, perfect measurement) DSGE model differ relates to the sizes of estimated standard deviations of the exogenous shocks driving business cycle in our model economy. From inspecting Table D3, one can observe that all standard deviations (except for σ_R) are getting smaller when we move from the regular to the data-rich case. In part, this is due to the fact that in the data-rich DSGE model we allow for the measurement error (or the theoretical gap between a particular model concept and a data indicator) so that a portion of fluctuations in all observables is accounted for by this indicator-specific component. This conclusion is further confirmed by inspecting Figure D1 that depicts the posterior means and 90 percent credible intervals for all three shocks – which are a subset of the DSGE state variables. As the figure shows, the estimated shocks in the data-rich DSGE model case seem to have smaller amplitude of fluctuations and are much smoother than their regular DSGE model counterparts.

As the sizes and the estimated time paths of exogenous shocks vary, the two models are also telling us quite different stories about the sources of business cycle fluctuations. When we assume the one-to-one data indicator – model concept correspondence and the perfect measurement, the four structural shocks are required to explain all fluctuations in the small 4–variable data set containing one measure of the real output, inflation, interest rate and the inverse money velocity. As we allow for multiple indicators and for the indicator-specific measurement error (or the theretical gap) and go for a richer data set, the results (see Table D5) suggest that the importance of some structural shocks may have been overstated.

Table D5 presents the unconditional variance decomposition of the core macro series for the regular and the data-rich DSGE models. Two overall conclusions stand out. First, the estimated indicator-specific measurement errors/theoretical gaps seem to account for a significant share of fluctuations in the core macro series considered, ranging from 4 to 82 percent. Second, as we move from the regular to the data-rich DSGE model, the role of technology innovations in generating fluctuations in real output, inflation and the interest rates is noticeably reduced.

Beginning with the real output, the diminished role of TFP shocks is partially compensated by the higher importance of the government spending shocks ranging from 10 to 17 percent. The increased role of the money demand shocks accounting for about 30 percent of unconditional variance of industrial production (IP) and IP: Manufacturing suggests that the IP's behavior over the business cycle is markedly different from that of the real GDP. From 2 to 4 percent of fluctuations in the measures of real output are due to the monetary policy innovations, a modest increase from 1 percent found in the regular DSGE model.

For the various theoretically distinct measures of inflation, the reduced role of TFP shocks is documented mostly on account of the non-negligible (19-36 percent) contribution of the idiosyncratic-specific component. In part, the lower contribution of technology innovations is taken over by the money demand shocks: they explain 3.1 – 3.5 percent of fluctuations in the PCE deflator inflation and the CPI inflation as compared to zero in the regular – perfect measurement, few observables – case.

Looking at the variance decomposition for the interest rates, we observe that the share of technology shocks has fallen from 96 percent in the regular to 67-82 percent in the data-rich DSGE model. The importance of the indicator-specific measurement error (theoretical gap) components, though, remained quite low. At the same time, we document a much higher contribution of the monetary policy innovations in generating fluctuations in interest rates. In the regular case – when the interest rate was assumed to be perfectly measured just by the federal funds rate – the monetary policy shocks accounted for only 4 percent of the unconditional variance. Once we allow for several noisy indicators of the interest rates, the contribution of the monetary policy shocks has risen to 14-17 percent.

When we assumed that the inverse money velocity is properly measured in the data by the single series – the IVM based on M2S aggregate, the major drivers of its fluctuations over the business cycle were the money demand shocks (about 60 percent) and technological innovations (29 percent), with contribution of the monetary policy shocks being essentially zero. After we moved to a data-rich environment and added to the list two measures of the IVM – one based on M1S aggregate and another based on the adjusted money base – the picture has changed dramatically. The role of the shocks to money demand has fallen considerably to 3 percent (IVM_MBase), 6 percent (IVM_M2S) and 17 percent (IVM_M1S), whereas the contribution to the unconditional variance of technology shocks has increased to 40 percent, though only for the inverse velocities based on M1 and monetary base. For the IVM_M2S, it is the indicator-specific "measurement error" that has become the major driver of fluctuations (82 percent) suggesting that our theoretical DSGE model captures the comovements in the real output and M2S balances quite poorly and is probably misspecified along this dimension. As expected, the results reveal a much greater role (10 percent) of the monetary policy in generating fluctuations of the IVM based on monetary base. This makes perfect sense given that the monetary base is the most fluid aggregate and is more interest-rate-sensitive than M1 and M2 aggregates.

E. Impulse Response Analysis

One of the most appealing features of DSGE models is that researchers and policymakers can use modern macroeconomic theory to interpret and predict the comovement of aggregate macro time series over the business cycle. Therefore, in this subsection we focus on propagating all structural innovations (government spending, money demand, monetary policy and technology) in both the regular DSGE model and the data-rich DSGE model with a view to generate and compare predictions for the key macroeconomic observables. By construction, in the regular DSGE model we are limited to obtain these predictions only for four primary series – real GDP, GDP deflator inflation, fed funds rate and real M2S, assumed to measure perfectly the corresponding model concepts. In the data-rich DSGE model, though, we could trace the dynamic effects of the same shocks to additional data indicators measuring real output, inflation, interest rates and real money balances. We defer the discussion of the impact of structural shocks on the non-core data variables in the data-rich DSGE model to the related work (Kryshko, 2011).

In Figure D2, we present the impulse response functions (IRFs) of the four primary macro observables: real GDP, GDP deflator inflation, fed funds rate and real M2S – to four one-standard-deviation structural shocks. A positive one-standard-deviation *government spending innovation* is associated with 60 to 80 basis points (b.p.) increase in real GDP on impact. Since the government finances its additional purchases through borrowing in the open market, it diverts part of the resources and partially "crowds out" private consumption and investment. Heavier borrowing raises nominal short-term interest rate by 2 to 5.5 b.p. and inhibits private investment even more, which in turn leads to declining return on capital and lower marginal costs. The latter explains the negative effect (15-30 b.p.) of g_t on GDP deflator inflation that we observe on impact. Finally, high interest rates raise the opportunity cost of holding money and households reduce their real money balances. As can be seen from Figure D2, the regular DSGE model clearly overstates the expansionary impact of government spending on real GDP by about 20 basis points and also overestimates the negative effect on GDP deflator inflation by 15 basis points (which is twice as the size of the effect in the data-rich DSGE model). At the same time, the impact of crowding out on the fed funds rate is clearly understated: the data-rich DSGE model predicts 5.5 b.p. increase at the 5th-quarter peak, while the regular DSGE model yields only 2 b.p. increase peaking in 2 years after the initial shock.

The second row of Figure D2 depicts the IRFs to the *money demand innovation*. It should be noted that in our theoretical New Keynesian model the money term enters the equilibrium conditions only in single place – in money demand equation (85). And the central bank is always assumed to supply enough money balances to satisfy all demand from households given current nominal interest rate. Because of that, the money balances are block exogenous and the money demand shocks – while raising or lowering M_t – do not affect either real output, or inflation or the interest rate in equilibrium. This is exactly true for the regular DSGE model, IRFs of which show positive response of the real M2S to one-std money demand shock and zero response of all other variables. This is approximately true in the data-rich DSGE model, but only for the four primary observables shown. The IRFs for the other noisy measures of real output, inflation, interest rate and real money balances (not shown) are non-zero and generally follow the patterns depicted by the thick blue line, though on a higher-scale grid: a positive money demand innovation raises real output contemporaneously, dampens prices and leads to the standard liquidity effect (lower interest rates associated with higher real money balances). The regular DSGE model differs from the data-rich one in that the former seems to overstate by a wide margin (roughly 45 basis points) the contemporaneous positive effect of the elevated money demand on real M2S.

Let us now turn to the effects of *monetary policy innovation*, which are summarized in the third row of Figure D2 and in Figure D3. A contractionary monetary policy shock corresponds to 60 (regular) – 75 (data-rich) basis points increase in the federal funds rate. Both versions of the DSGE models predict that the real GDP and the GDP deflator inflation will fall by 40-50 b.p. and 25-30 b.p., respectively, before returning to their trend paths. As the nominal policy rate rises and the opportunity costs of holding money for households increase, we observe a strong liquidity effect associated with falling real money balances (50 b.p. in the regular and 72 b.p. in the data-rich DSGE model). Also, high interest rates make the saving motive and buying more bonds temporarily a more attractive option. This raises households' marginal utility of consumption and discourages current spending in favor of the future consumption. Because the household faces investment adjustment costs and cannot adjust investment quickly, and government spending in the model is exogenous, the lower consumption leads to a fall in aggregate demand. The firms respond to lower demand in part by contracting real output and in part by reducing the optimal price. Hence, the aggregate price level falls, but not as much given nominal rigidities in the intermediate goods-producing sector. Notice that despite some on-impact differences, the responses of all variables to the monetary policy innovation remain very similar and quantitatively close in the regular and the data-rich DSGE models.

The real challenge is revealed in Figure D3. The IRFs of the other measures of the real output and inflation to the monetary policy innovation produce puzzling results. For example, industrial production: total and industrial production: manufacturing actually rise following a contractionary monetary policy shock, at least on impact. By the same token, the PCE deflator inflation and CPI inflation react positively to monetary tightening, despite GDP deflator inflation – the primary inflation measure – responding negatively as prescribed by theory. We discuss further the potential reasons for that and show how to deal with these puzzling results in Kryshko (2011). For now, we would just like to note that these puzzles may indicate the potential misspecification of our DSGE model.

We plot the effects of a positive *technology innovation* in row 4 of Figure D2 and in Figure D4 (other core series). Following positive TFP shock, the real GDP broadly increases, as our economy becomes more productive and the firms find it optimal to produce more. Both models generate the hump-shaped positive IRFs; the regular DSGE model predicts that the maximal impact on real GDP of 75 basis points is achieved at the 14th-quarter peak, while the data-rich DSGE model's response is more persistent, but is twice as low and peaks roughly at the 23rd quarter. New demand come primarily from higher capital investment, reflecting much better future return on capital, and also from additional household consumption fueled by greater income. The higher output on the supply side plus improved efficiency implies a downward pressure on prices (GDP deflator inflation falls by 52 basis points in the data-rich versus 90 basis points in the regular DSGE model). The increase in real GDP above steady state and the fall of inflation below target level, under the estimated monetary policy Taylor rule, requires the Fed to move the policy rate in opposite directions. The fact that the Fed actually lowers the policy rate means that the falling prices effect dominates. Declining interest rate boosts real output even more, which in turn raises further the return on capital. As the positive impact of technological innovation dissipates, this higher return, through the future marginal costs channel, fuels inflationary expectations that ultimately translate into contemporaneous upward price pressures. The Fed reacts by increasing the policy rate, which explains the observed hump in the fed funds interest rate IRF. Given temporarily lower interest rates, households choose to hold, with some lag, relatively higher real money balances. A part of the growing money demand comes endogenously from the elevated level of economic activity. A general observation from comparing the IRFs from the regular and the data-rich DSGE models is that the regular DSGE model tends to overestimate all effects of TFP shocks, though on impact they might not be too different.

Looking at the responses of the alternative measures of real output, inflation, interest rates and real money balances to the positive TFP shock (Figure D4), we generally conclude that they remain qualitatively similar to the reactions of primary data indicators and we don't observe puzzles as documented above for the effects of monetary tightening. The measures of industrial production tend to rise, although more slowly than the real GDP, the price inflations tend to fall though the magnitude of the on-impact effect is twice as low. The 3-month T-bill rate and the AAA bond yield broadly follow the path of the federal funds rate, with bond yield falling slower and lagging roughly 4 quarters. The measures of real money balances respond by and large positively and with a hump, yet the initial responses of the real M1S and the real monetary base remain negative for two quarters in a row.

VI. Conclusions

In a growing body of literature that estimates macroeconomic DSGE models, two assumptions remain very common: (i) that a particular model concept is perfectly measured by a single data series without an error, and (ii) that all relevant information to estimate the state and the parameters of the economy is summarized by a few observable data indicators, usually equal to the number of structural shocks in the model. In this paper, we relaxed these two assumptions and estimated a version of the monetary DSGE model with standard New Keynesian core on a richer data set. This so called *data-rich DSGE model* can be seen as a combination of a regular DSGE model and a dynamic factor model in which factors are the economic state variables of the DSGE model and the transition of factors is governed by a DSGE model solution.

We used the post-1983 U.S. data on real output, inflation, nominal interest rates, measures of inverse money velocity and a large panel of the other informational macroeconomic and financial series to estimate and compare the new data-rich DSGE model with a regular – few observables, perfect measurement – DSGE model, both sharing the same theoretical core. The estimation involved Bayesian MCMC methods. Because of the data set's high panel dimension, the likelihood-based estimation of the data-rich DSGE model was computationally very challenging. To reduce the costs, we employed a novel speedup as in Jungbacker and Koopman (2008) and achieved the computational time savings of 60 percent.

We documented that the data-rich DSGE model generates a higher duration of the Calvo price contracts and a lower implied slope of the New Keynesian Phillips curve measuring the elasticity of current inflation to real marginal costs. As we moved from the regular to the data-rich DSGE model, we found that: (i) the role of technology innovations in generating fluctuations in real output, inflation and the interest rates is noticeably reduced; and that (ii) the contribution of monetary policy shocks to cyclical fluctuations in interest rates increased from 4 to 14-17 percent. Regarding dynamic propagation, we established that (i) despite some slight on-impact differences, the responses of all primary observables (real GDP, GDP deflator inflation, fed funds rate and real M2) to the monetary policy innovation remain theoretically plausible and quantitatively close in the regular and in the data-rich DSGE models; and that (ii) the regular DSGE model tended to overestimate all effects of TFP shocks, though on impact they might not have been too different. Finally, we found some puzzling results for the responses of industrial production, the PCE deflator inflation and the CPI inflation to monetary tightening, which may indicate the potential misspecification of our theoretical DSGE model. We plan to address and discuss these issues and puzzles in further research.

Appendix A. DSGE Model

Appendix A1. First-Order Conditions of Household

The problem of each household j is to maximize the utility function (9) subject to budget constraint (11) and capital accumulation equation (12) for all t. Associate Lagrange multipliers $\lambda_t(j)$ and $Q_t(j)$ with constraints (11) and (12), respectively. Then, the First Order Conditions with respect to $x_t(j)$, $h_t(j)$, $m_{t+1}(j)$, $i_t(j)$, $k_{t+1}(j)$ and $b_{t+1}(j)$ are:

$$\lambda_t(j) = \frac{U'(x_t(j))}{P_t} \tag{59}$$

$$\lambda_t(j) = \frac{A}{P_t W_t} \tag{60}$$

$$\frac{U'(x_t(j))}{P_t} = \beta E_t \left\{ \frac{\chi_{t+1}}{P_{t+1}} \left(\frac{A}{Z_*^{1/(1-\alpha)}} \right)^{(1-\nu_m)} \left(\frac{m_{t+1}(j)}{P_{t+1}} \right)^{-\nu_m} + \frac{U'(x_{t+1}(j))}{P_{t+1}} \right\} \tag{61}$$

$$\begin{aligned} 1 = \mu_t(j) \left[1 - S\left(\frac{i_t(j)}{i_{t-1}(j)} \right) - S'\left(\frac{i_t(j)}{i_{t-1}(j)} \right) \frac{i_t(j)}{i_{t-1}(j)} \right] + \\ + \beta E_t \left\{ \mu_{t+1}(j) \frac{U'(x_{t+1}(j))}{U'(x_t(j))} S'\left(\frac{i_{t+1}(j)}{i_t(j)} \right) \left[\frac{i_{t+1}(j)}{i_t(j)} \right]^2 \right\} \end{aligned} \tag{62}$$

$$\mu_t(j) = \beta E_t \left\{ \frac{U'(x_{t+1}(j))}{U'(x_t(j))} \left(R_{t+1}^k + \mu_{t+1}(j)(1-\delta) \right) \right\} \tag{63}$$

$$1 = \beta E_t \left\{ \frac{U'(x_{t+1}(j))}{U'(x_t(j))} \frac{R_t}{\pi_{t+1}} \right\}, \tag{64}$$

where $\pi_t = P_t / P_{t-1}$ denotes inflation and where we have substituted out Lagrange multiplier $\lambda_t(j)$ with its equivalent expression using marginal utility of consumption and have introduced the normalized shadow price of installed capital $\mu_t(j) = \dfrac{Q_t(j)}{U'(x_t(j))}$.

We do not take first order conditions with respect to A-D securities holdings $a_{t+1}(j)$ explicitly, because we make use of the result due to Erceg, Henderson and Levin (2000). This result says that under the assumption of complete markets for A-D securities and under the additive separability of labor and money balances in household's utility, the equilibrium price of A-D securities will be such that optimal consumption will not depend on idiosyncratic shocks. Hence, all households will share the same marginal utility of consumption, and given (59), Lagrange multiplier $\lambda_t(j)$ will also be the same across all households: $\lambda_t(j) = \lambda_t$, all j and t. This implies that in equilibrium all households will choose the same consumption, money and bond holdings, investment and capital. Note that we don't have wage rigidity in this model – therefore the choice of optimal labor will also be same. This implies that we can safely drop index j from all equilibrium conditions of households and proceed accordingly.

The first two FOCs could be combined to yield labor supply equation relating real wage to marginal rate of substitution between consumption and labor. (61) is an Euler equation for money holdings, which together with (64) – an Euler equation for bond holdings – implies household's optimal demand for real money balances. Equation (62) determines the law of motion for shadow price of installed capital. If there were no investment adjustment costs, this price will be equal to 1, which is standard in neoclassical growth model. Also note that if we were to have an investment specific technology shock, this shadow price will be equal to relative price of capital in consumption units. Equation (63) is an Euler equation for capital holdings. The shadow cost of purchasing one unit of capital today should be equal to the real return from renting it to firms plus the tomorrow's resale value of capital that has not yet depreciated.

Appendix A2. First-Order Conditions of Intermediate Goods Firm

Monopolistically competitive intermediate goods producer i, which is allowed to re-optimize, chooses the optimal price $P_t^o(i)$ that maximizes discounted stream of profits subject to optimal demand from final good producers:

$$\begin{aligned} &\max_{P_t^o(i)} \quad E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi_{t+s|t}^{p}\left(P_t^o(i)\pi_{t+s|t}^{adj}-P_{t+s}MC_{t+s}\right)Y_{t+s}(i)\right\} \\ &\text{s.t.} \quad Y_{t+s}(i)=\left[\frac{P_t^o(i)\pi_{t+s|t}^{adj}}{P_{t+s}}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{t+s}, \qquad s=0,1,2,\ldots. \end{aligned} \tag{65}$$

First, obtain an expression for $\dfrac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}$:

$$\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}=-\frac{(1+\lambda)}{\lambda}\left[\frac{P_t^o(i)\pi_{t+s|t}^{adj}}{P_{t+s}}\right]^{-\frac{(1+\lambda)}{\lambda}-1}\frac{\pi_{t+s|t}^{adj}}{P_{t+s}}Y_{t+s}=-\left(\frac{1+\lambda}{\lambda}\right)\frac{Y_{t+s}(i)}{P_t^o(i)}. \tag{66}$$

Now the first order condition for the problem (65), where we will plug optimal demand $Y_{t+s}(i)$ into the objective function and assume interior solution, is:

$$E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi_{t+s|t}^{p}\left[\pi_{t+s|t}^{adj}\left(Y_{t+s}(i)+P_t^o(i)\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}\right)-P_{t+s}MC_{t+s}\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}\right]\right\}=0. \tag{67}$$

Consider expression inside square brackets:

$$[\cdots] \quad = \pi^{adj}_{t+s|t} Y_{t+s}(i) + \left(P^o_t(i)\pi^{adj}_{t+s|t} - P_{t+s} MC_{t+s} \right) \frac{\partial Y_{t+s}(i)}{\partial P^o_t(i)} =$$

$$= \pi^{adj}_{t+s|t} Y_{t+s}(i) - \left(P^o_t(i)\pi^{adj}_{t+s|t} - P_{t+s} MC_{t+s} \right) \frac{(1+\lambda)}{\lambda} \frac{Y_{t+s}(i)}{P^o_t(i)} =$$

$$= \frac{Y_{t+s}(i)}{P^o_t(i)} \left(P^o_t(i)\pi^{adj}_{t+s|t} - \frac{(1+\lambda)}{\lambda} \left(P^o_t(i)\pi^{adj}_{t+s|t} - P_{t+s} MC_{t+s} \right) \right) =$$

$$= \frac{1}{\lambda} \frac{Y_{t+s}(i)}{P^o_t(i)} \left((\lambda - 1 - \lambda) P^o_t(i)\pi^{adj}_{t+s|t} + (1+\lambda) P_{t+s} MC_{t+s} \right).$$

Cancelling out $1/\lambda \neq 0$ and multiplying (67) by -1, we could rewrite the FOC as follows:

$$E_t \left\{ \sum_{s=0}^{\infty} (\zeta\beta)^s \Xi^p_{t+s|t} \frac{Y_{t+s}(i)}{P^o_t(i)} \left[P^o_t(i)\pi^{adj}_{t+s|t} - (1+\lambda) P_{t+s} MC_{t+s} \right] \right\} = 0. \qquad (68)$$

Remark 1: Since for $s > 0$, $\pi^{adj}_{t+s|t} = \prod_{l=1}^{s} \pi^{\iota}_{t+l-1} \pi^{(1-\iota)}_{**}$ and $\pi^{adj}_{(t+1)+s|(t+1)} = \prod_{l=1}^{s} \pi^{\iota}_{(t+1)+l-1} \pi^{(1-\iota)}_{**} =$ $= \pi^{\iota}_{t+1}\pi^{\iota}_{t+2}...\pi^{\iota}_{t+s}\pi^{(1-\iota)s}_{**}$, it follows that $\pi^{adj}_{t+(s+1)|t} = \prod_{l=1}^{s+1} \pi^{\iota}_{t+l-1}\pi^{(1-\iota)}_{**} = \pi^{\iota}_t \pi^{\iota}_{t+1}\pi^{\iota}_{t+2}...\pi^{\iota}_{t+s}\pi^{(1-\iota)(s+1)}_{**} =$ $= \left[\pi^{\iota}_t \pi^{(1-\iota)}_{**} \right] \pi^{adj}_{(t+1)+s|(t+1)}$.

Remark 2: Since for $s > 0$, $\Xi^p_{t+s|t} = \frac{\lambda_{t+s}}{\lambda_t}$ and so $\Xi^p_{t+(s+1)|t} = \frac{\lambda_{t+s+1}}{\lambda_t}$, it follows that $\Xi^p_{(t+1)+s|(t+1)} = \frac{\lambda_{t+1+s}}{\lambda_{t+1}} \frac{\lambda_t}{\lambda_t} = = \Xi^p_{t+(s+1)|t} \big/ \Xi^p_{t+1|t}$ and that $\Xi^p_{t+(s+1)|t} = \Xi^p_{t+1|t} \Xi^p_{(t+1)+s|(t+1)}$.

Remark 3: Notice that given expression for an optimal demand for good i in (65), $Y_{t+(s+1)}(i) \neq Y_{(t+1)+s}(i)$. However, using result from Remark 1, we obtain:

$$Y_{t+(s+1)}(i) = \left[\frac{P^o_{t+1}(i)}{P^o_{t+1}(i)} \frac{P^o_t(i)\pi^{adj}_{t+(s+1)|t}}{P_{t+(s+1)}} \right]^{-\frac{(1+\lambda)}{\lambda}} Y_{t+(s+1)} =$$

$$= \left[\frac{P^o_t(i)}{P^o_{t+1}(i)} \right]^{-\frac{(1+\lambda)}{\lambda}} \left[\frac{P^o_{t+1}(i)}{P_{(t+1)+s}} \left[\pi^{\iota}_t \pi^{(1-\iota)}_{**} \right] \pi^{adj}_{(t+1)+s|(t+1)} \right]^{-\frac{(1+\lambda)}{\lambda}} Y_{t+1+s} =$$

$$= \left[\frac{P^o_t(i)}{P^o_{t+1}(i)} \right]^{-\frac{(1+\lambda)}{\lambda}} \left[\pi^{\iota}_t \pi^{(1-\iota)}_{**} \right]^{-\frac{(1+\lambda)}{\lambda}} Y_{(t+1)+s}(i)$$

To express FOC (68) recursively, we define two auxiliary variables:

$$f^{(1)}_t = E_t \left\{ \sum_{s=0}^{\infty} (\zeta\beta)^s \Xi^p_{t+s|t} Y_{t+s}(i) \pi^{adj}_{t+s|t} \right\} \qquad (69)$$

$$f_t^{(2)} = E_t \left\{ \sum_{s=0}^{\infty} (\zeta\beta)^s \Xi_{t+s|t}^p Y_{t+s}(i) \frac{P_{t+s}}{P_t^o(i)} MC_{t+s} \right\}, \tag{70}$$

so that FOC becomes:

$$f_t^{(1)} = (1+\lambda) f_t^{(2)}. \tag{71}$$

Recalling that $\Xi_{t|t}^p = 1$, $\pi_{t|t}^{adj} = 1$ and using results from Remarks 1, 2 and 3, we can rewrite (69) as:

$$\begin{aligned} f_t^{(1)} &= Y_t(i) + E_t \left\{ \sum_{k=0}^{\infty} (\zeta\beta)^{k+1} \Xi_{t+(k+1)|t}^p Y_{t+(k+1)}(i) \pi_{t+(k+1)|t}^{adj} \right\} = \\ &= Y_t(i) + (\zeta\beta) E_t \left\{ \sum_{k=0}^{\infty} (\zeta\beta)^k (\Xi_{t+1|t}^p \Xi_{(t+1)+k|(t+1)}^p) \left[\frac{P_t^o(i)}{P_{t+1}^o(i)} \right]^{-\frac{(1+\lambda)}{\lambda}} \times \right. \\ &\quad \left. \times \left[\pi_t^\iota \pi_{**}^{(1-\iota)} \right]^{-\frac{(1+\lambda)}{\lambda}} Y_{(t+1)+k}(i) \left[\pi_t^\iota \pi_{**}^{(1-\iota)} \right] \pi_{(t+1)+k|(t+1)}^{adj} \right\} = \\ &= Y_t(i) + \zeta\beta \left[\pi_t^\iota \pi_{**}^{(1-\iota)} \right]^{-\frac{1}{\lambda}} E_t \left\{ \left[\frac{P_t^o(i)}{P_{t+1}^o(i)} \right]^{-\frac{(1+\lambda)}{\lambda}} \Xi_{t+1|t}^p \sum_{k=0}^{\infty} (\zeta\beta)^k \Xi_{(t+1)+k|(t+1)}^p Y_{(t+1)+k}(i) \pi_{(t+1)+k|(t+1)}^{adj} \right\} = \\ &= \left[\frac{P_t^o(i)}{P_t} \right]^{-\frac{(1+\lambda)}{\lambda}} Y_t + \zeta\beta \left[\pi_t^\iota \pi_{**}^{(1-\iota)} \right]^{-\frac{1}{\lambda}} E_t \left\{ \left[\frac{P_t^o(i)}{P_{t+1}^o(i)} \right]^{-\frac{(1+\lambda)}{\lambda}} \Xi_{t+1|t}^p f_{t+1}^{(1)} \right\}. \end{aligned} \tag{72}$$

Similarly, the recursion for $f_t^{(2)}$ becomes:

$$\begin{aligned} f_t^{(2)} &= Y_t(i) \frac{P_t MC_t}{P_t^o(i)} + (\zeta\beta) E_t \left\{ \sum_{k=0}^{\infty} (\zeta\beta)^k (\Xi_{t+1|t}^p \Xi_{(t+1)+k|(t+1)}^p) \left[\frac{P_t^o(i)}{P_{t+1}^o(i)} \right]^{-\frac{(1+\lambda)}{\lambda}} \times \right. \\ &\quad \left. \times \left[\pi_t^\iota \pi_{**}^{(1-\iota)} \right]^{-\frac{(1+\lambda)}{\lambda}} Y_{(t+1)+k}(i) \frac{P_{t+1+k}}{P_t^o(i)} MC_{t+1+k} \frac{P_{t+1}^o(i)}{P_{t+1}^o(i)} \right\} = \\ &= \left[\frac{P_t^o(i)}{P_t} \right]^{-\frac{(1+\lambda)}{\lambda}-1} MC_t Y_t + \zeta\beta \left[\pi_t^\iota \pi_{**}^{(1-\iota)} \right]^{-\frac{(1+\lambda)}{\lambda}} E_t \left\{ \left[\frac{P_t^o(i)}{P_{t+1}^o(i)} \right]^{-\frac{(1+\lambda)}{\lambda}-1} \Xi_{t+1|t}^p f_{t+1}^{(2)} \right\}. \end{aligned} \tag{73}$$

In summary, the first order conditions of the problem (27) boil down to these three equations:

$$f_t^{(1)} = \left(p_t^o \right)^{-\frac{(1+\lambda)}{\lambda}} Y_t + \zeta\beta \left(\pi_t^\iota \pi_{**}^{(1-\iota)} \right)^{-\frac{1}{\lambda}} E_t \left\{ \left(\frac{p_t^o}{p_{t+1}^o \pi_{t+1}} \right)^{-\frac{(1+\lambda)}{\lambda}} \Xi_{t+1|t}^p f_{t+1}^{(1)} \right\} \tag{74}$$

$$f_t^{(2)} = \left(p_t^o\right)^{-\frac{(1+\lambda)}{\lambda}-1} MC_t Y_t + \zeta\beta\left(\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{(1+\lambda)}{\lambda}} E_t\left\{\left(\frac{p_t^o}{p_{t+1}^o\pi_{t+1}}\right)^{-\frac{(1+\lambda)}{\lambda}-1} \Xi_{t+1|t}^p f_{t+1}^{(2)}\right\} \tag{75}$$

$$f_t^{(1)} = (1+\lambda) f_t^{(2)}, \tag{76}$$

where we have defined the optimal price relative to the price level $p_t^o = \frac{P_t^o}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$.

Appendix A3. Evolution of Price Dispersion

Aggregate price dispersion across intermediate goods firms is captured by variable $D_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} di$. By properties of Calvo pricing, $P_t(i)$ is equal to optimal price P_t^o with probability $1-\zeta$ (optimizing firms) and is equal to $\left[\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right]P_{t-1}(i)$ with probability ζ (non-optimizing firms). Therefore, by definition of D_t we have:

$$D_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} di = (1-\zeta)\left(\frac{P_t^o}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} + \zeta\left[\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} \int_0^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} di =$$

$$= (1-\zeta)\left(\frac{P_t^o}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} + \zeta\left[\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} \int_0^1 \left(\frac{P_{t-1}(i)}{P_{t-1}}\right)^{-\frac{(1+\lambda)}{\lambda}} di$$

The last line implies:

$$D_t = (1-\zeta)\left(\frac{P_t^o}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} + \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} D_{t-1} \tag{77}$$

Appendix A4. Equilibrium Conditions and Aggregate Disturbances

We define equilibrium in our economy in a standard way. It is determined by the optimality conditions and laws of motion summarized below:

(1) Households' optimality conditions

$$U'(x_t) = \frac{A}{W_t} \tag{78}$$

$$\frac{U'(x_t)}{P_t} = \beta E_t\left\{\frac{\chi_{t+1}}{P_{t+1}}\left(\frac{A}{Z_*^{1/(1-\alpha)}}\right)^{(1-\nu_m)}\left(\frac{m_{t+1}}{P_{t+1}}\right)^{-\nu_m} + \frac{U'(x_{t+1})}{P_{t+1}}\right\} \tag{79}$$

$$1 = \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) - S'\left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} S'\left(\frac{i_{t+1}}{i_t} \right) \left[\frac{i_{t+1}}{i_t} \right]^2 \right\} \tag{80}$$

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left(R^k_{t+1} + \mu_{t+1}(1-\delta) \right) \right\} \tag{81}$$

$$1 = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right\} \tag{82}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right] i_t \tag{83}$$

$$\Xi^p_{t+1|t} = \frac{U'(x_{t+1})}{U'(x_t)} \frac{1}{\pi_{t+1}} \tag{84}$$

Note that (79) and (82) imply money demand equation[12]:

$$\left(\bar{M}_t \right)^{\nu_m} = \left(\frac{m_{t+1}}{P_t} \right)^{\nu_m} = \frac{\beta R_t}{U'(x_t)(R_t - 1)} E_t \left\{ \left(\frac{A}{Z_*^{1/(1-\alpha)}} \right)^{(1-\nu_m)} \frac{\chi_{t+1}}{\pi_{t+1}^{(1-\nu_m)}} \right\}. \tag{85}$$

(2) Firms' optimality conditions

$$K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R^k_t} H_t \tag{86}$$

$$MC_t = \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \frac{W_t^{1-\alpha} \left(R^k_t \right)^{\alpha}}{Z_t} \tag{87}$$

$$f^{(1)}_t = \left(p^o_t \right)^{-\frac{(1+\lambda)}{\lambda}} Y_t + \zeta\beta \left(\pi^{\iota}_t \pi^{(1-\iota)}_{**} \right)^{-\frac{1}{\lambda}} E_t \left\{ \left(\frac{p^o_t}{p^o_{t+1}\pi_{t+1}} \right)^{-\frac{(1+\lambda)}{\lambda}} \Xi^p_{t+1|t} f^{(1)}_{t+1} \right\} \tag{88}$$

$$f^{(2)}_t = \left(p^o_t \right)^{-\frac{(1+\lambda)}{\lambda}-1} MC_t Y_t + \zeta\beta \left(\pi^{\iota}_t \pi^{(1-\iota)}_{**} \right)^{-\frac{(1+\lambda)}{\lambda}} E_t \left\{ \left(\frac{p^o_t}{p^o_{t+1}\pi_{t+1}} \right)^{-\frac{(1+\lambda)}{\lambda}-1} \Xi^p_{t+1|t} f^{(2)}_{t+1} \right\} \tag{89}$$

$$f^{(1)}_t = (1+\lambda) f^{(2)}_t \tag{90}$$

$$\pi_t = \left[(1-\zeta) \left(\pi_t p^o_t \right)^{-\frac{1}{\lambda}} + \zeta \left(\pi^{\iota}_{t-1} \pi^{(1-\iota)}_{**} \right)^{-\frac{1}{\lambda}} \right]^{-\lambda}, \tag{91}$$

where we have denoted $p^o_t = P^o_t / P_t$ and where equilibrium requires $K_t = k_t$, $H_t = h_t$.

[12] We deflate nominal money stock m_{t+1} by P_t (and not P_{t+1}) since it has been chosen in period *t* based on realization of period *t* disturbances. We denote corresponding real money balances by $\bar{M}_{t+1} = m_{t+1}/P_t$.

(3) Taylor rule

$$\frac{R_t}{R_*}=\left(\frac{R_{t-1}}{R_*}\right)^{\rho_R}\left(\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1}\left(\frac{Y_t}{Y_*}\right)^{\psi_2}\right)^{(1-\rho_R)} e^{\varepsilon_{R,t}}, \qquad \text{where } \varepsilon_{R,t}\sim N(0,\sigma_R^2) \tag{92}$$

(4) Aggregate demand and supply

$$X_t+I_t+\left(1-\frac{1}{g_t}\right)Y_t=Y_t \tag{93}$$

$$Y_t=\frac{1}{D_t}(Z_tK_t^{\alpha}H_t^{1-\alpha}-\tilde{F}) \tag{94}$$

where equilibrium requires that $X_t=x_t$ and $I_t=i_t$, and that:

$$D_t=\zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} D_{t-1}+(1-\zeta)\left[p_t^o\right]^{-\frac{(1+\lambda)}{\lambda}}. \tag{95}$$

(5) Aggregate disturbances (technology, money demand, government spending and monetary policy):

$$\ln Z_t=(1-\rho_Z)\ln Z_*+\rho_Z\ln Z_{t-1}+\varepsilon_{Z,t} \tag{96}$$
$$\ln \chi_t=(1-\rho_\chi)\ln \chi_*+\rho_\chi\ln \chi_{t-1}+\varepsilon_{\chi,t} \tag{97}$$
$$\ln g_t=(1-\rho_g)\ln g_*+\rho_g\ln g_{t-1}+\varepsilon_{g,t}, \tag{98}$$

where it is understood that innovations to the above laws of motion, as well as the monetary policy shock $\varepsilon_{R,t}$, are $iid\ N(0,\sigma_i^2)$ random variables, $i\in\{Z,\chi,g,R\}$.

Appendix A5. Steady State and Log-Linearized Equilibrium Conditions

In what follows we specialize the household's utility to be constant-relative-risk-aversion function:

$$U(x_t)=B\frac{x_t^{1-\gamma}}{1-\gamma}.$$

In addition, for any generic variable V_t the corresponding "star" variable V_* denotes its steady state value and "hat" variable stands for log-deviation from steady state: $\hat{V}_t=\ln(V_t/V_*)$

Steady State Conditions

$$R_*=\frac{\pi_*}{\beta}$$

$$R_*^k = \frac{1}{\beta} + \delta - 1$$

$$p_*^o = \left(\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta} \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{1-\iota}{\lambda}} \right)^{-\lambda}$$

$$D_* = \frac{1-\zeta}{1-\zeta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1+\lambda)}{\lambda}(1-\iota)}} \left(p_*^o \right)^{-\frac{(1+\lambda)}{\lambda}}$$

$$\overline{Y}_* = Y_* D_*$$

$$K_* = \frac{\alpha p_*^o (\overline{Y}_* + \tilde{F})}{(1+\lambda) R_*^k} \left(\frac{1-\zeta\beta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1+\lambda)}{\lambda}(1-\iota)}}{1-\zeta\beta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1-\iota)}{\lambda}}} \right)$$

$$Z_* = \frac{(\overline{Y}_* + \tilde{F})}{K_*^{\alpha} H_*^{1-\alpha}}$$

$$I_* = \delta K_*$$

$$W_* = \frac{1-\alpha}{\alpha} \frac{K_*}{H_*} R_*^k$$

$$X_* + I_* + \left(1 - \frac{1}{g_*} \right) Y_* = Y_*$$

$$A = \frac{1}{\overline{M}_*} \left(\frac{\chi_* W_* \pi_*^{\nu_m}}{(R_* - 1) Z_*^{\frac{1-\nu_m}{1-\alpha}}} \right)^{\frac{1}{\nu_m}}$$

$$B X_*^{-\gamma} = \frac{A}{W_*}$$

Log-Linearized Equilibrium Conditions

Households

$$\hat{W}_t = \gamma \hat{X}_t$$

$$\hat{I}_t = \frac{1}{1+\beta} \hat{I}_{t-1} + \frac{\beta}{1+\beta} \hat{I}_{t+1} + \frac{1}{S''(1+\beta)} \hat{\mu}_t$$

$$-\gamma \hat{X}_t = -\gamma \hat{X}_{t+1} + (\hat{R}_t - \hat{\pi}_{t+1})$$

$$\hat{\mu}_t - \gamma \hat{X}_t = \beta(1-\delta) \hat{\mu}_{t+1} - \gamma \hat{X}_{t+1} + \beta R_*^k \hat{R}_{t+1}^k$$

$$\hat{K}_{t+1} = (1-\delta) \hat{K}_t + \delta \hat{I}_t$$

$$\hat{\Xi}_{t|t-1}^p = -\gamma (\hat{X}_t - \hat{X}_{t-1}) - \hat{\pi}_t$$

$$\nu_m \hat{\hat{M}}_{t+1} = \gamma \hat{X}_t + \nu_m \hat{\chi}_{t+1} - (1-\nu_m)\hat{\pi}_{t+1} - \frac{1}{R_* - 1}\hat{R}_t$$

Firms

$$\hat{K}_t = \hat{H}_t + \hat{W}_t - \hat{R}_t^k$$

$$\hat{MC}_t = (1-\alpha)\hat{W}_t + \alpha \hat{R}_t^k - \hat{Z}_t$$

$$\hat{f}_t^{(1)} = \hat{f}_t^{(2)}$$

$$\hat{f}_t^{(1)} = (1-C_1)\left(-\frac{1+\lambda}{\lambda}\hat{p}_t^o + \hat{Y}_t\right) + C_1\left(-\frac{\iota}{\lambda}\hat{\pi}_t + \frac{1+\lambda}{\lambda}[-\hat{p}_t^o + \hat{\pi}_{t+1} + \hat{p}_{t+1}^o] + \hat{\Xi}_{t+1|t}^p + \hat{f}_{t+1}^{(1)}\right)$$

$$\hat{f}_t^{(2)} = (1-C_2)\left(-\left(\frac{1+\lambda}{\lambda}+1\right)\hat{p}_t^o + \hat{Y}_t + \hat{MC}_t\right) +$$

$$C_2\left(-\frac{\iota(1+\lambda)}{\lambda}\hat{\pi}_t + \left(\frac{1+\lambda}{\lambda}+1\right)[-\hat{p}_t^o + \hat{\pi}_{t+1} + \hat{p}_{t+1}^o] + \hat{\Xi}_{t+1|t}^p + \hat{f}_{t+1}^{(2)}\right)$$

$$\hat{p}_t^o = (C_3 - 1)\hat{\pi}_t - C_3 \iota \zeta \left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}} \hat{\pi}_{t-1},$$

where $C_1 = \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}$, $C_2 = \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1+\lambda}{\lambda}(1-\iota)}$, $C_3 = \frac{1}{1-\zeta}\left(p_*^o\right)^{\frac{1}{\lambda}}$.

Taylor Rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_1 \hat{\pi}_t + \psi_2 \hat{Y}_t) + \varepsilon_{R,t}$$

Aggregate Demand and Supply

$$\hat{\hat{Y}}_t = \hat{Y}_t + \hat{D}_t$$

$$\hat{\hat{Y}}_t = \left(1 + \frac{\tilde{F}}{\bar{\bar{Y}}_*}\right)(\hat{Z}_t + \alpha \hat{K}_t + (1-\alpha)\hat{H}_t)$$

$$\hat{D}_t = \left(-\frac{p_*^0}{D_*}\frac{1+\lambda}{\lambda}(1-\zeta)\right)\hat{p}_t^o + \zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1+\lambda}{\lambda}(1-\iota)}\left(\hat{D}_{t-1} + \frac{1+\lambda}{\lambda}\hat{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\hat{\pi}_{t-1}\right)$$

$$\hat{Y}_t = \frac{X_*}{X_* + I_*}\hat{X}_t + \frac{I_*}{X_* + I_*}\hat{I}_t + \hat{g}_t$$

Aggregate Disturbances

$$\hat{Z}_t = \rho_Z \hat{Z}_{t-1} + \varepsilon_{Z,t}, \qquad \varepsilon_{Z,t} \sim iid\ N(0, \sigma_Z^2)$$

$$\hat{\chi}_t = \rho_\chi \hat{\chi}_{t-1} + \varepsilon_{\chi,t}, \qquad \varepsilon_{\chi,t} \sim iid\ N(0, \sigma_\chi^2)$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t}, \qquad \varepsilon_{g,t} \sim iid\ N(0, \sigma_g^2)$$

$$\varepsilon_{R,t} \sim iid\ N(0, \sigma_R^2)$$

APPENDIX B. DETAILS OF MARKOV CHAIN MONTE CARLO ALGORITHM

Appendix B1. Data-Rich DSGE Model: Gibbs Sampler: Step 2.2.a): Generating Unobserved States S^T

To sample the unobserved states S^T from $p(S^T \mid \Gamma, \boldsymbol{\theta}; X^T)$, given the state-space model parameters Γ and the structural DSGE model parameters $\boldsymbol{\theta}$, we will use the Carter-Kohn (1994) forward-backward algorithm. We begin by quasi-differencing the measurement equation (41)

$$X_t = \boldsymbol{\Lambda}(\boldsymbol{\theta})S_t + e_t \tag{99}$$

to obtain the *iid* normal errors: $(\mathbf{I} - \boldsymbol{\Psi}L)X_t = (\mathbf{I} - \boldsymbol{\Psi}L)\boldsymbol{\Lambda}(\boldsymbol{\theta})S_t + v_t$. Since the matrix polynomial multiplying S_t is of order 1, we can stack the additional lag of S_t and rewrite our linear Gaussian state-space system as follows:

$$\tilde{X}_t = \underbrace{\left[\boldsymbol{\Lambda}(\boldsymbol{\theta}) \;\vert\; -\boldsymbol{\Psi}\boldsymbol{\Lambda}(\boldsymbol{\theta})\right]}_{\tilde{\boldsymbol{\Lambda}}} \cdot \begin{bmatrix} S_t \\ S_{t-1} \end{bmatrix} + v_t \tag{100}$$

$$\underbrace{\begin{bmatrix} S_t \\ S_{t-1} \end{bmatrix}}_{\tilde{S}_t} = \underbrace{\begin{bmatrix} \mathbf{G}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{G}}} \underbrace{\begin{bmatrix} S_{t-1} \\ S_{t-2} \end{bmatrix}}_{\tilde{S}_{t-1}} + \underbrace{\begin{bmatrix} \mathbf{H}(\boldsymbol{\theta}) \\ \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{H}}} \varepsilon_t, \tag{101}$$

or more compactly:

$$\tilde{X}_t = \tilde{\boldsymbol{\Lambda}}\tilde{S}_t + v_t \tag{102}$$

$$\tilde{S}_t = \tilde{\mathbf{G}}\tilde{S}_{t-1} + \tilde{\mathbf{H}}\varepsilon_t \tag{103}$$

where $\tilde{X}_t = X_t - \boldsymbol{\Psi}X_{t-1}$, $v_t \sim iid\ N(\mathbf{0}, \mathbf{R})$, and $\varepsilon_t \sim iid\ N(\mathbf{0}, \mathbf{Q}(\boldsymbol{\theta}))$. For convenience, collect all the parameter matrices in $\Xi = \{\tilde{\boldsymbol{\Lambda}}, \mathbf{R}, \tilde{\mathbf{G}}, \tilde{\mathbf{H}}, \mathbf{Q}(\boldsymbol{\theta})\}$.

As in Carter-Kohn (1994), we first apply the Kalman filter to the state-space system (102)-(103) to generate the filtered DSGE states $\tilde{S}_{t|t}$ and their covariance matrices $\tilde{\mathbf{P}}_{t|t}$, for $t = 1..T$ (forward pass of the algorithm):

$$\text{prediction} \quad \begin{cases} \tilde{S}_{t+1|t} = \tilde{\mathbf{G}}\tilde{S}_{t|t} \\ \tilde{\mathbf{P}}_{t+1|t} = \tilde{\mathbf{G}}\tilde{\mathbf{P}}_{t|t}\tilde{\mathbf{G}}' + (\tilde{\mathbf{H}}\mathbf{Q}(\boldsymbol{\theta})\tilde{\mathbf{H}}') \\ \eta_{t+1|t} = \tilde{X}_t - \tilde{\boldsymbol{\Lambda}}\tilde{S}_{t+1|t} \\ f_{t+1|t} = \tilde{\boldsymbol{\Lambda}}\tilde{\mathbf{P}}_{t+1|t}\tilde{\boldsymbol{\Lambda}}' + \mathbf{R} \end{cases} \tag{104}$$

$$\text{updating} \quad \begin{cases} \tilde{S}_{t+1|t+1} = \tilde{S}_{t+1|t} + \mathbf{K}_t\eta_{t+1|t} \\ \tilde{\mathbf{P}}_{t+1|t+1} = \tilde{\mathbf{P}}_{t+1|t} - \mathbf{K}_t\tilde{\boldsymbol{\Lambda}}\tilde{\mathbf{P}}_{t+1|t} \end{cases} \tag{105}$$

where $\mathbf{K}_t = \tilde{\mathbf{P}}_{t+1|t}\tilde{\mathbf{\Lambda}}' f_{t+1|t}^{-1}$ is the Kalman gain and $\eta_{t+1|t}$ is the period *t* prediction error. Second, starting from $\tilde{S}_{T|T}$ and $\tilde{\mathbf{P}}_{T|T}$, we roll back in time and draw the elements of S^T from a sequence of conditional Gaussian distributions. We draw $\tilde{S}_T$ from its conditional distribution given parameters Ξ and data $\tilde{X}^T$

$$\tilde{S}_T \mid \Xi, \tilde{X}^T \sim N(\tilde{S}_{T|T}, \tilde{\mathbf{P}}_{T|T}). \tag{106}$$

We generate $\tilde{S}_t$ for $t = T-1, T-2, ..., 1$ by proceeding backwards and by drawing from

$$\tilde{S}_t \mid \tilde{S}^*_{t+1}, \Xi, \tilde{X}^t \sim N(\tilde{S}_{t|t,\tilde{S}^*_{t+1}}, \tilde{\mathbf{P}}_{t|t,\tilde{S}^*_{t+1}}), \tag{107}$$

where $\tilde{X}^t = \left\{\tilde{X}_1, ..., \tilde{X}_t\right\}$ and

$$\tilde{S}_{t|t,\tilde{S}^*_{t+1}} = \tilde{S}_{t|t} + \tilde{\mathbf{P}}_{t|t}\tilde{\mathbf{G}}^{*\prime}\left[\tilde{\mathbf{G}}^*\tilde{\mathbf{P}}_{t|t}\tilde{\mathbf{G}}^{*\prime} + \mathbf{Q}^*\right]^{-1}\left(\tilde{S}^*_{t+1} - \tilde{\mathbf{G}}^*\tilde{S}_{t|t}\right) \tag{108}$$

$$\tilde{\mathbf{P}}_{t|t,\tilde{S}^*_{t+1}} = \tilde{\mathbf{P}}_{t|t} - \tilde{\mathbf{P}}_{t|t}\tilde{\mathbf{G}}^{*\prime}\left[\tilde{\mathbf{G}}^*\tilde{\mathbf{P}}_{t|t}\tilde{\mathbf{G}}^{*\prime} + \mathbf{Q}^*\right]^{-1}\tilde{\mathbf{G}}^*\tilde{\mathbf{P}}_{t|t}. \tag{109}$$

Notice that the covariance matrix $\mathbf{\Sigma}_u$ of the error term $u_t = \tilde{\mathbf{H}}\varepsilon_t$ in state transition equation (103) is singular:

$$\mathbf{\Sigma}_u = E(u_t u_t') = E(\tilde{\mathbf{H}}\varepsilon_t\varepsilon_t'\tilde{\mathbf{H}}') = \left[\begin{array}{c|c} \mathbf{H}(\boldsymbol{\theta})\mathbf{Q}(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\theta})' & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} \end{array}\right] \tag{110}$$

Therefore, we use the approach of Kim and Nelson (1999b, p. 194-196) and condition the distribution of $\tilde{S}_t$ on only a non-identity-related part of $\tilde{S}_{t+1}$ (namely $\tilde{S}^*_{t+1}$) that corresponds to the non-singular upper-left corner of $\mathbf{\Sigma}_u$ (otherwise, if we conditioned on full state vector $\tilde{S}_{t+1}$, we would be unable to draw $\tilde{S}_t$, since the covariance matrix in (107) would be singular). This requires that

$$\tilde{S}^*_{t+1} = \mathbf{M}\tilde{S}_{t+1}, \quad \tilde{\mathbf{G}}^* = \mathbf{M}\tilde{\mathbf{G}}, \quad \mathbf{Q}^* = \mathbf{M}\mathbf{\Sigma}_u\mathbf{M}', \tag{111}$$

where $\mathbf{M}$ is the appropriate selection matrix consisting of 0s and 1s.

To initialize the Kalman filter (104)-(105), we set $\tilde{S}_{0|0}$ and $\tilde{\mathbf{P}}_{0|0}$ to the unconditional mean and covariance of the DSGE states $\tilde{S}_t$.

Appendix B2. Data-Rich DSGE Model: Gibbs Sampler: Step 2.2.b): Generating State-Space Parameters Γ

To sample the state-space parameters $\Gamma = \{\mathbf{\Lambda}, \mathbf{R}, \mathbf{\Psi}\}$ from $p(\Gamma \mid S^T, \boldsymbol{\theta}; X^T)$ given the unobserved DSGE states S^T and the structural DSGE model parameters $\boldsymbol{\theta}$, we use the approach of Chib and Greenberg (1994). Due to diagonality of $\mathbf{R}$ and $\mathbf{\Psi}$, and conditional on known unobserved states S^T, the equations (41)-(42) represent a collection of the linear regressions with AR(1) errors, with k^{th} equation given by

$$X_{k,t} = \mathbf{\Lambda}_k' S_t + e_{k,t} \tag{112}$$

$$e_{k,t} = \Psi_{kk} e_{k,t-1} + v_{k,t}, \qquad v_{k,t} \sim iid \;\; N(0, R_{kk}) \tag{113}$$

where $\boldsymbol{\Lambda}'_k$ is a $1 \times N$ vector and a k^{th} row of $\boldsymbol{\Lambda}$. Therefore in what follows we will draw the elements in Γ equation by equation for $k = 1..J$.

For each $\left(\boldsymbol{\Lambda}_k, R_{kk}, \Psi_{kk}\right)$, we consider the following conjugate prior distribution:

$$\begin{aligned} &p(\boldsymbol{\Lambda}_k, R_{kk}, \Psi_{kk}) = p(\boldsymbol{\Lambda}_k, R_{kk})\, p(\Psi_{kk}) = \\ &= NIG_2(\boldsymbol{\Lambda}_k, R_{kk} \mid \boldsymbol{\Lambda}_{k,0}; \mathbf{M}_{k,0}; s_0; \nu_0) \times N(\Psi_{kk} \mid \Psi_0, \sigma^2_{\Psi,0}) \mathbf{1}_{\{|\Psi_{kk}|<1\}}, \end{aligned} \tag{114}$$

in which we set the parameters of Normal-Inverse-Gamma-2 density to $s_0 = 0.001$, $\nu_0 = 3$ and $\boldsymbol{\Lambda}_{k,0}$, $\mathbf{M}_{k,0}$ may in general depend on $\boldsymbol{\theta}$, and where we take $\Psi_0 = 0$ and $\sigma^2_{\Psi,0} = 1$.

Conditional posterior density of $\left(\boldsymbol{\Lambda}_k, \boldsymbol{R}_{kk}\right)$: The posterior density is of the form

$$p(\boldsymbol{\Lambda}_k, R_{kk} \mid \Psi_{kk}; S^T, \boldsymbol{\theta}, X^T) \propto p(X_k^T \mid S^T, \boldsymbol{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta})\, p(\boldsymbol{\Lambda}_k, R_{kk}) . \tag{115}$$

Define

$$X^*_{k,t} = X_{k,t} - \Psi_{kk} X_{k,t-1} \qquad S^*_t = S_t - \Psi_{kk} S_{t-1} \tag{116}$$

and rewrite (112)-(113) as a linear regression:

$$X^*_{k,t} = \boldsymbol{\Lambda}'_k S^*_t + v_{k,t} . \tag{117}$$

Define $T \times 1$ matrix $X^*_k = [X^*_{k,1}, X^*_{k,2}, ..., X^*_{k,T}]'$ and $T \times N$ matrix $S^* = [S^*_1, S^*_2, ..., S^*_T]'$ and rewrite (117) in matrix form:

$$X^*_k = S^* \boldsymbol{\Lambda}_k + v_k \tag{118}$$

It can be shown (Chib, Greenberg 1994, Bauwens, Lubrano, Richard 1999, Theorem 2.22, p. 57) that the likelihood of (118) is proportional to a Normal-Inverse-Gamma-2 density defined as

$$p(X_k^T \mid S^T, \boldsymbol{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta}) \propto p_{NIG_2}(\boldsymbol{\Lambda}_k, R_{kk} \mid \hat{\boldsymbol{\Lambda}}_k, (S^{*\prime} S^*), s, T - N - 2) , \tag{119}$$

where[13]

$$\hat{\boldsymbol{\Lambda}}_k = \left(S^{*\prime} S^*\right)^{-1} S^{*\prime} X^*_k \tag{120}$$

$$s = X^{*\prime}_k \left(\mathbf{I}_T - S^* \left(S^{*\prime} S^*\right)^{-1} S^{*\prime} \right) X^*_k = X^{*\prime}_k \left(X^*_k - S^* \hat{\boldsymbol{\Lambda}}_k \right) \tag{121}$$

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[13] Normalization constant in p_{NIG_2} is $C_{Ng}(\mathbf{M}, s, \nu; p) = \text{Gamma}\left(\frac{\nu}{2}\right)\left(\frac{2}{s}\right)^{\nu/2} (2\pi)^{p/2} |\mathbf{M}|^{-\frac{1}{2}}$, where $p = \dim \boldsymbol{\beta}$.

$$p_{NIG_2}(\boldsymbol{\beta},\sigma^2 \mid \boldsymbol{\mu},\, \mathbf{M},\, s,\, \nu) = C_{Ng}^{-1}(\mathbf{M},\, s,\, \nu;\, p)\left(\sigma^2\right)^{-\frac{1}{2}(\nu+p+2)} \times \\ \times\ \exp\left\{-\frac{1}{2\sigma^2}\left[s+(\boldsymbol{\beta}-\boldsymbol{\mu})'\mathbf{M}(\boldsymbol{\beta}-\boldsymbol{\mu})\right]\right\} \tag{122}$$

Since the assumed prior $p(\boldsymbol{\Lambda}_k, R_{kk})$ is also of Normal-Inverse-Gamma-2 form, by Theorem 2.24 (Bauwens, Lubrano, Richard 1999, p. 56-61) we deduce:

$$\begin{aligned} p(\boldsymbol{\Lambda}_k, R_{kk} \mid \Psi_{kk}; S^T, \boldsymbol{\theta}, X^T) &\propto p_{NIG_2}(\boldsymbol{\Lambda}_k, R_{kk} \mid \hat{\boldsymbol{\Lambda}}_k,\, (S^{*\prime}S^*),\, s,\, T-N-2)\times \\ &\times\ p_{NIG_2}(\boldsymbol{\Lambda}_k, R_{kk} \mid \boldsymbol{\Lambda}_{k,0}, \mathbf{M}_{k,0}, s_0, \nu_0) \\ &\propto p_{NIG_2}(\boldsymbol{\Lambda}_k, R_{kk} \mid \bar{\boldsymbol{\Lambda}}_k, \bar{\mathbf{M}}_k, \bar{s}, \bar{\nu}), \end{aligned} \tag{123}$$

with parameters given by

$$\bar{\mathbf{M}}_k = \mathbf{M}_{k,0} + \left(S^{*\prime}S^*\right)$$

$$\bar{\boldsymbol{\Lambda}}_k = \bar{\mathbf{M}}_k^{-1}\left(\mathbf{M}_{k,0}\boldsymbol{\Lambda}_{k,0} + (S^{*\prime}S^*)\hat{\boldsymbol{\Lambda}}_k\right)$$

$$\bar{s} = s_0 + s + \left(\boldsymbol{\Lambda}_{k,0} - \hat{\boldsymbol{\Lambda}}_k\right)'\left[\mathbf{M}_{k,0}^{-1} + \left(S^{*\prime}S^*\right)^{-1}\right]^{-1}\left(\boldsymbol{\Lambda}_{k,0} - \hat{\boldsymbol{\Lambda}}_k\right)$$

$$\bar{\nu} = \nu_0 + T\ .$$

The alternative equivalent expression for $\bar{s}$ used in computations is

$$\bar{s} = s_0 + s + \boldsymbol{\Lambda}_{k,0}'\mathbf{M}_{k,0}\boldsymbol{\Lambda}_{k,0} + \hat{\boldsymbol{\Lambda}}_k'\left(S^{*\prime}S^*\right)\hat{\boldsymbol{\Lambda}}_k - \bar{\boldsymbol{\Lambda}}_k'\bar{\mathbf{M}}_k\bar{\boldsymbol{\Lambda}}_k$$

The resulting conditional posterior density of $\left(\boldsymbol{\Lambda}_k, R_{kk}\right)$ is Normal-Inverse-Gamma-2, and we sample the loadings $\boldsymbol{\Lambda}_k$ and the variance of measurement error R_{kk} sequentially from:

$$\begin{aligned} R_{kk} \mid \Psi_{kk}; S^T, \boldsymbol{\theta}, X^T &\sim IG_2(\bar{s}, \bar{\nu}) \\ \boldsymbol{\Lambda}_k \mid R_{kk}, \Psi_{kk}; S^T, \boldsymbol{\theta}, X^T &\sim N_N(\bar{\boldsymbol{\Lambda}}_k, R_{kk}\bar{\mathbf{M}}_k^{-1}) \end{aligned} \tag{124}$$

Conditional posterior density of Ψ_{kk}: The posterior density is of the form

$$p(\Psi_{kk} \mid \boldsymbol{\Lambda}_k, R_{kk}; S^T, \boldsymbol{\theta}, X^T) \propto p(X_k^T \mid S^T, \boldsymbol{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta})\, p(\Psi_{kk}) \tag{125}$$

Similar to what we did above, we define

$$e_{k,t} = X_{k,t} - \boldsymbol{\Lambda}_k' S_t \qquad e_k = \begin{bmatrix} e_{k,2} \\ \vdots \\ e_{k,T} \end{bmatrix} \qquad e_{k,-1} = \begin{bmatrix} e_{k,1} \\ \vdots \\ e_{k,T-1} \end{bmatrix} \tag{126}$$

and rewrite (113) in matrix form:

$$e_k = e_{k,-1}\Psi_{kk} + v_k \tag{127}$$

Because now we only care about the autocorrelation parameter Ψ_{kk}, the likelihood function in (125) is proportional to the normal density

$$\begin{aligned} p(X_k^T \mid S^T, \mathbf{\Lambda}_k, R_{kk}, \Psi_{kk}, \mathbf{\theta}) &\propto \exp\left\{-\frac{1}{2R_{kk}}(e_k - e_{k,-1}\Psi_{kk})'(e_k - e_{k,-1}\Psi_{kk})\right\} \\ &\propto \exp\left\{-\frac{1}{2R_{kk}}(\Psi_{kk} - \hat{\Psi}_{kk})'\left(e'_{k,-1}e_{k,-1}\right)(\Psi_{kk} - \hat{\Psi}_{kk})\right\} \end{aligned} \tag{128}$$

with $\hat{\Psi}_{kk} = \left(e'_{k,-1}e_{k,-1}\right)^{-1} e'_{k,-1}e_k$. Provided that the prior for Ψ_{kk} is truncated normal with mean Ψ_0 and variance $\sigma^2_{\Psi,0}$, the conditional posterior density is proportional to a product of two normals:

$$\begin{aligned} p(\Psi_{kk} \mid \mathbf{\Lambda}_k, R_{kk}; S^T, \mathbf{\theta}, X^T) &\propto \exp\left\{-\frac{1}{2R_{kk}}(\Psi_{kk} - \hat{\Psi}_{kk})'\left(e'_{k,-1}e_{k,-1}\right)(\Psi_{kk} - \hat{\Psi}_{kk})\right\} \\ &\propto \exp\left\{-\frac{1}{2\sigma^2_{\Psi,0}}(\Psi_{kk} - \Psi_0)^2\right\} \times \mathbf{1}_{\{|\Psi_{kk}|<1\}} \end{aligned}$$

This implies that the conditional posterior of Ψ_{kk} is (truncated) normal $N(\bar{\Psi}_{kk}, \bar{V}_{\Psi_{kk}}) \times \mathbf{1}_{\{|\Psi_{kk}|<1\}}$ with

$$\begin{aligned} \bar{V}_{\Psi_{kk}} &= \left(\left[R_{kk}\left(e'_{k,-1}e_{k,-1}\right)^{-1}\right]^{-1} + \left(\sigma^2_{\Psi,0}\right)^{-1}\right)^{-1} \\ \bar{\Psi}_{kk} &= \bar{V}_{\Psi_{kk}}\left(\left[R_{kk}\left(e'_{k,-1}e_{k,-1}\right)^{-1}\right]^{-1}\hat{\Psi}_{kk} + \left(\sigma^2_{\Psi,0}\right)^{-1}\Psi_0\right) \end{aligned} \tag{129}$$

APPENDIX C. DATA: DESCRIPTION AND TRANSFORMATIONS

#	Short Name	SW Mnemonic	Trans Code	Description
Core Series				
	Real Output			
1.	RGDP		4	Real Per-capita Gross Domestic Product
2.	IP_TOTAL		4	Per-capita Industrial Production Index: Total
3.	IP_MFG		4	Per-capita Industrial Production Index: Manufacturing
	Inflation			
4.	PGDP		4	GDP Deflator Inflation
5.	PCED		4	Personal Consumption Expenditure Deflator Inflation
6.	CPI_ALL		4	Consumer Price Index (All Items) Inflation
	Nominal Interest Rate			
7.	FedFunds		4	Interest Rate: Federal Funds (effective), % per annum
8.	TBill_3m		4	Interest Rate: U.S. Treasury bills, secondary market, 3 month, % per annum
9.	AAABond		4	Bond Yield: Moody's AAA Corporate, % per annum
	Inverse Velocity of Money (M/Y)			
10.	IVM_M1S_det		4	Inverse Velocity of Money based on M1S aggregate
11.	IVM_M2S		4	Inverse Velocity of Money based on M2S aggregate
12.	IVM_MBase_bar		4	Inverse Velocity of Money based on adjusted Monetary Base
Non-Core Series				
	Output and Components			
1.	IP_CONS_DBLE	IPS13	3*	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
2.	IP_CONS_NONDBLE	IPS18	3*	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
3.	IP_BUS_EQPT	IPS25	3*	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
4.	IP_DBLE_MATS	IPS34	3*	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
5.	IP_NONDBLE_MATS	IPS38	3*	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
6.	IP_FUELS	IPS306	3*	INDUSTRIAL PRODUCTION INDEX - FUELS
7.	PMP	PMP	0	NAPM PRODUCTION INDEX (PERCENT)
8.	RCONS	GDP252	3*	Real Personal Consumption Expenditures, Quantity Index (2000=100) , SAAR
9.	RCONS_DUR	GDP253	3*	Real Personal Consumption Expenditures - Durable Goods , Quantity Index (2000=100), SAAR
10.	RCONS_SERV	GDP255	3*	Real Personal Consumption Expenditures - Services, Quantity Index (2000=100) , SAAR
11.	REXPORTS	GDP263	3*	Real Exports, Quantity Index (2000=100) , SAAR
12.	RIMPORTS	GDP264	3*	Real Imports, Quantity Index (2000=100) , SAAR
13.	RGOV	GDP265	3*	Real Government Consumption Expenditures & Gross Investment, Quantity Index (2000=100), SAAR
	Labor Market			
14.	EMP_MINING	CES006	3*	EMPLOYEES, NONFARM - MINING
15.	EMP_CONST	CES011	3*	EMPLOYEES, NONFARM - CONSTRUCTION
16.	EMP_DBLE_GDS	CES017	3*	EMPLOYEES, NONFARM - DURABLE GOODS
17.	EMP_NONDBLES	CES033	3*	EMPLOYEES, NONFARM - NONDURABLE GOODS
18.	EMP_SERVICES	CES046	3*	EMPLOYEES, NONFARM - SERVICE-PROVIDING
19.	EMP_TTU	CES048	3*	EMPLOYEES, NONFARM - TRADE, TRANSPORT, UTILITIES
20.	EMP_WHOLESALE	CES049	3*	EMPLOYEES, NONFARM - WHOLESALE TRADE
21.	EMP_RETAIL	CES053	3*	EMPLOYEES, NONFARM - RETAIL TRADE
22.	EMP_FIRE	CES088	3	EMPLOYEES, NONFARM - FINANCIAL ACTIVITIES
23.	EMP_GOVT	CES140	3	EMPLOYEES, NONFARM - GOVERNMENT
24.	URATE_ALL	LHUR	0	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
25.	U_DURATION	LHU680	0	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
26.	U_L5WKS	LHU5	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
27.	U_5_14WKS	LHU14	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
28.	U_M15WKS	LHU15	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
29.	U_15_26WKS	LHU26	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
30.	U_M27WKS	LHU27	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
31.	HOURS_AVG	CES151	0	AVG WKLY HOURS, PROD WRKRS, NONFARM - GOODS-PRODUCING
	Housing			
32.	HSTARTS_NE	HSNE	1	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
33.	HSTARTS_MW	HSMW	1	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
34.	HSTARTS_SOU	HSSOU	1	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
35.	HSTARTS_WST	HSWST	1	HOUSING STARTS:WEST (THOUS.U.)S.A.

35.	HSTARTS_WST	HSWST	1	HOUSING STARTS:WEST (THOUS.U.)S.A.
36.	RRESINV	GDP261	3*	Real Gross Private Domestic Investment - Residential, Quantity Index (2000=100), SAAR
	Financial Variables			
37.	SFYGM6	Sfygm6	0	fygm6-fygm3 fygm6: INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA) fygm3: INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
38.	SFYGT1	Sfygt1	0	fygt1-fygm3 fygt1: INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
39.	SFYGT10	Sfygt10	0	fygt10-fygm3 fygt10: INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
40.	SFYBAAC	sFYBAAC	0	FYBAAC-Fygt10 FYBAAC: BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
41.	BUS_LOANS	BUSLOANS	3	Commercial and Industrial Loans at All Commercial Banks (FRED) Billions $ (SA)
42.	CONS_CREDIT	CCINRV	3*	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
43.	DLOG_EXR_US	EXRUS	2	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
44.	DLOG_EXR_CHF	EXRSW	2	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)
45.	DLOG_EXR_YEN	EXRJAN	2	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)
46.	DLOG_EXR_GBP	EXRUK	2	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
47.	DLOG_EXR_CAN	EXRCAN	2	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
48.	DLOG_SP500	FSPCOM	2	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
49.	DLOG_SP_IND	FSPIN	2	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
50.	DLOG_DJIA	FSDJ	2	COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE
	Investment, Inventories, Orders			
51.	NAPMI	PMI	0	PURCHASING MANAGERS' INDEX (SA)
52.	NAPM_NEW_ORDRS	PMNO	0	NAPM NEW ORDERS INDEX (PERCENT)
53.	NAPM_VENDOR_DEL	PMDEL	0	NAPM VENDOR DELIVERIES INDEX (PERCENT)
54.	NAPM_INVENTORIES	PMNV	0	NAPM INVENTORIES INDEX (PERCENT)
55.	RINV_GDP	GDP256	3*	Real Gross Private Domestic Investment, Quantity Index (2000=100) , SAAR
56.	RNONRESINV_STRUCT	GDP259	1	Real Gross Private Domestic Investment - Nonresidential - Structures, Quantity Index (2000=100), SAAR
57.	RNONRESINV_BEQUIPT	GDP260	3*	Real Gross Private Domestic Investment - Nonresidential - Equipment & Software
	Prices and Wages			
58.	RAHE_CONST	CES277R	3*	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION (CES277/PI071)
59.	RAHE_MFG	CES278R	3	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG (CES278/PI071)
60.	P_COM	PSCCOMR	2	Real SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100) (PSCCOM/PCEPILFE) PSCCOM: SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100) PCEPILFE: PCE Price Index Less Food and Energy (SA) Fred
61.	P_OIL	PW561R	2	PPI Crude (Relative to Core PCE) (pw561/PCEPiLFE) pw561: PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)
62.	P_NAPM_COM	PMCP	2	NAPM COMMODITY PRICES INDEX (PERCENT)
63.	RCOMP_HOUR	LBPUR7	1*	REAL COMPENSATION PER HOUR,EMPLOYEES:NONFARM BUSINESS(82=100,SA)
64.	ULC	LBLCPU	1*	UNIT LABOR COST: NONFARM BUSINESS SEC (1982=100,SA)
65.	PCED_DUR	GDP274A	2	Personal Consumption Expenditures: Durable goods Price Index
66.	PCED_NDUR	GDP275A	2	Personal Consumption Expenditures: Nondurable goods Price Index
67.	PCED_SERV	GDP276A	2	Personal Consumption Expenditures: Services Price Index
68.	PINV_GDP	GDP277A	2	Gross private domestic investment Price Index
69.	PINV_NRES_STRUCT	GDP280A	2	GPDI Price Index: Structures
70.	PINV_NRES_EQP	GDP281A	2	GPDI Price Index: Equipment and software Price Index
71.	PINV_RES	GDP282A	2	GPDI Price Index: Residential Price Index
72.	PEXPORTS	GDP284A	2	GDP: Exports Price Index
73.	PIMPORTS	GDP285A	2	GDP: Imports Price Index
74.	PGOV	GDP286A	2	Government consumption expenditures and gross investment Price Index
	Other			
75.	UTL11	UTL11	0	CAPACITY UTILIZATION - MANUFACTURING (SIC)
76.	UMICH_CONS	HHSNTN	1	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)
77.	LABOR_PROD	LBOUT	1*	OUTPUT PER HOUR ALL PERSONS: BUSINESS SEC(1982=100,SA)

Notes: Transformation codes: 0 – nothing; 1 – log(); 2 – dlog(); 3 – log of the ratio of subaggregate to aggregate; 4 – transformation described in the main text, pp. 19. Asterisk (*) indicates the transformed variable has been further linearly detrended.

Source of data: Stock and Watson (2008), "Forecasting in Dynamic Factor Models Subject to Structural Instability," available online at http://www.princeton.edu/~mwatson/ddisk/hendryfestschrift_replicationfiles_April28_2008.zip

Full sample available: 1959:Q1-2006:Q4. Sample used in estimation: 1984:Q1-2005:Q4.

All series available at monthly frequency have been converted to quarterly by simple averaging in native units.

APPENDIX D. TABLES AND FIGURES

Table D1. Data-Rich DSGE Model: Parameters Fixed During Estimation - Calibration and Normalization

Parameter Name	**Mnemonics**	**Value**
Depreciation rate	δ	0.014
Risk aversion in HH utility function	γ	1
Money demand shock in steady state	χ_*	1
Share of govt spending in steady state	g_*	1.2
Fixed costs in production	F	0
MP rule: response to inflation	ψ_1	1.82
MP rule: response to output gap	ψ_2	0.18
MP rule: int rate smoothing parameter	ρ_R	0.78
Persistence: TFP shock	ρ_Z	0.98
Steady state inflation (in % pa)	π_A	2.5
Steady state real interest rate (in % pa)	r_A	2.84
Price indexation parameter	π_{**}	1
Steady state real GDP	Y_*	1
Log inverse velocity of money in SS	$\log(M_* / Y_*)$	0.778
Steady state of log average inverse labor productivity	$\log(H_* / Y_*)$	–3.5
Transformations: $\beta = \dfrac{1}{1 + r_A/400}$; $\pi_* = 1 + \dfrac{\pi_A}{400}$		

Table D2. Data-Rich DSGE Model: Prior Distributions

Parameter Name		Domain	Density	Para 1	Para 2
Firms					
Share of capital	α	[0;1)	Beta	0.3	0.025
Average economy wide markup	λ	*R+*	Gamma	0.15	0.01
$1-\zeta$ prob of reoptimizing firm's price	ζ	[0;1)	Beta	0.6	0.15
Indexation parameter	ι	[0;1)	Beta	0.5	0.25
Households					
Elasticity of money demand	ν_m	*R+*	Gamma	20	5
Investment adjustment cost parameter	S''	*R+*	Gamma	5.0	2.5
Shocks					
Persistence: govt spending process	ρ_g	[0;1)	Beta	0.8	0.1
Persistence: money demand shock	ρ_χ	[0;1)	Beta	0.8	0.1
Stdev: govt spending process	σ_g	*R+*	InvGamma	1	4
Stdev: money demand shock	σ_χ	*R+*	InvGamma	1	4
Stdev: monetary policy shock	σ_R	*R+*	InvGamma	0.5	4
Stdev: TFP shock	σ_Z	*R+*	InvGamma	1	4

Notes: Para 1 and Para 2 are (i) the means and the standard deviations for Beta, Gamma, and Normal distributions; (ii) the upper and the lower bound of support for the Uniform distribution; (iii) s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma \mid s,\nu) \propto \sigma^{-\nu-1}\exp(-\nu s^2/2\sigma^2)$.

Table D3. Data-Rich DSGE Model: Posterior Estimates

Parameter Name		Regular DSGE model Mean	Regular DSGE model 90% CI	Data-Rich DSGE model Mean	Data-Rich DSGE model 90% CI
Firms					
Share of capital	α	0.282	[0.269, 0.296]	0.2766	[0.266, 0.292]
Average economy wide markup	λ	0.15	[0.133, 1.166]	0.134	[0.117, 0.154]
$1-\zeta$ prob of reoptimizing firm's price	ζ	0.759	[0.709, 0.809]	0.797	[0.777, 0.819]
Indexation parameter	ι	0.05	[0.00, 0.101]	0.0326	[0.001, 0.0636]
Households					
Elasticity of money demand	ν_m	25.943	[19.581, 31.65]	23.199	[17.13, 31.27]
Investment adjustment cost parameter	S''	11.079	[6.299, 15.683]	30.754	[26.506, 35.29]
Shocks					
Persistence: govt spending process	ρ_g	0.886	[0.85, 0.92]	0.870	[0.839, 0.909]
Persistence: money demand shock	ρ_χ	0.974	[0.958, 0.992]	0.961	[0.936, 0.981]
Stdev: govt spending process	σ_g	1.227	[1.062, 1.388]	0.851	[0.605, 1.238]
Stdev: money demand shock	σ_χ	0.865	[0.757, 0.972]	0.396	[0.327, 0.464]
Stdev: monetary policy shock	σ_R	0.199	[0.175, 0.223]	0.2404	[0.211, 0.275]
Stdev: TFP shock	σ_Z	0.557	[0.471, 0.639]	0.375	[0.322, 0.439]
Implied Slope of NK Phillips Curve	κ	0.0745		0.0517	

Notes: Results labeled **"Regular DSGE model"** refer to the standard Bayesian estimation of the same underlying theoretical DSGE model as presented in the main text, but only on 4 core observable data series (real GDP, GDP deflator inflation, the federal funds rate and the inverse velocity of money based on the M2S aggregate) assumed to be perfectly measured. In terms of the state-space representation (40)-(42), this means that the vector of data X_t contains just these 4 core observables, the factor loadings $\mathbf{\Lambda}$ are restricted as below, and there are no measurement errors e_t:

$$\underbrace{\begin{bmatrix} \text{RealGDP}_t \\ \text{GDP\_Def\_Inflation}_t \\ \text{FedFundsRate}_t \\ \text{IVM\_M2S}_t \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 4 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 4 & 0 & \cdots & 0 \\ -1 & 0 & 0 & 1 & \cdots & 0 \end{bmatrix}}_{\mathbf{\Lambda}} \cdot \underbrace{\begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ \hat{M}_t \\ \vdots \end{bmatrix}}_{S_t} \quad (130)$$

Table D4. Data-Rich DSGE Model: Summary of the Unconditional Variance Decomposition

iid Measurement Errors; Dataset = DFM3.txt
on average, 20K draws, 4K burn-in

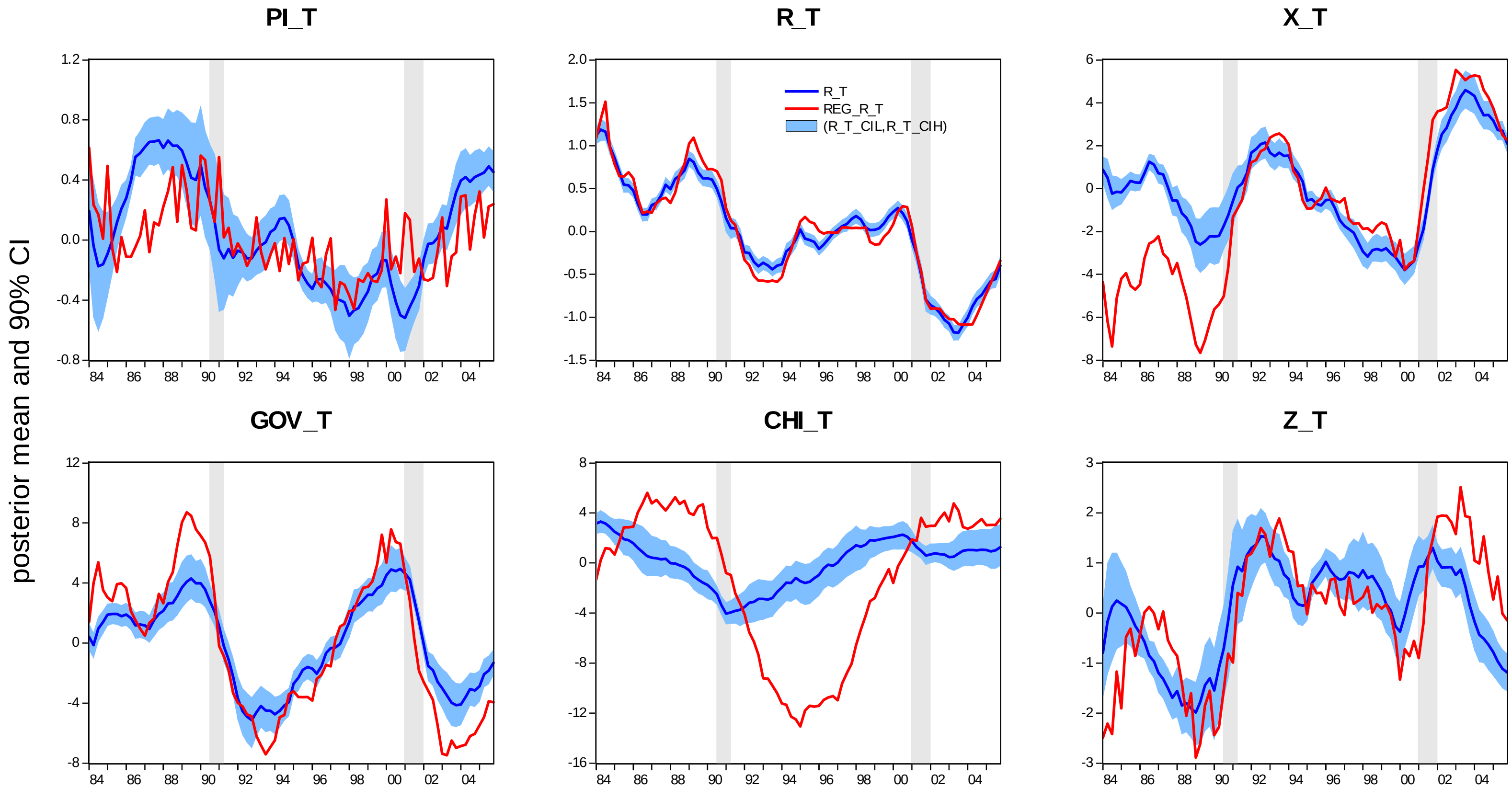
	GOV	CHI	MP	Z	All Shocks	Error term
Core Variables	**0.05**	**0.08**	**0.06**	**0.56**	**0.749**	**0.251**
Real output	0.14	0.21	0.03	0.48	0.852	0.148
Inflation	0.01	0.02	0.01	0.70	0.733	0.267
Interest rates	0.01	0.00	0.15	0.76	0.925	0.075
Money velocities	0.07	0.09	0.04	0.29	0.489	0.512
Non-Core Variables	**0.09**	**0.13**	**0.06**	**0.45**	**0.719**	**0.281**
Output and components	0.07	0.27	0.08	0.45	0.873	0.127
Labor market	0.19	0.14	0.06	0.46	0.848	0.152
Investment, inventories, orders	0.10	0.13	0.02	0.63	0.882	0.118
Housing	0.04	0.26	0.07	0.42	0.794	0.206
Prices and wages	0.03	0.05	0.04	0.45	0.568	0.432
Financial variables	0.06	0.03	0.05	0.32	0.451	0.549
Other	0.02	0.12	0.09	0.64	0.866	0.134

Table D5. Data-Rich DSGE vs. Regular DSGE Model: Unconditional Variance Decomposition

		GOV	CHI	MP	Z	All Shocks	Measurement Error
Regular DSGE:	**Real GDP**	0.099	0.000	0.012	0.889	1.000	-
Data-Rich DSGE:	**Real GDP**	0.081	0.000	0.040	0.648	0.770	0.230
	IP Total	0.167	0.308	0.021	0.395	0.891	0.110
	IP Manufacturing	0.166	0.317	0.020	0.392	0.894	0.106
Regular DSGE:	**GDP Def inflation**	0.020	0.000	0.009	0.970	1.000	-
Data-Rich DSGE:	**GDP Def inflation**	0.011	0.000	0.011	0.789	0.811	0.189
	PCE Def inflation	0.004	0.035	0.003	0.703	0.745	0.255
	CPI ALL Inflation	0.005	0.031	0.006	0.600	0.642	0.358
Regular DSGE:	**Fed Funds rate**	0.001	0.000	0.040	0.959	1.000	-
Data-Rich DSGE:	**Fed Funds rate**	0.004	0.000	0.135	0.817	0.956	0.044
	3m T-Bill rate	0.007	0.003	0.160	0.788	0.958	0.042
	AAA Bond yield	0.013	0.008	0.168	0.672	0.861	0.139
Regular DSGE:	**IVM_M2S**	0.117	0.596	0.001	0.286	1.000	-
Data-Rich DSGE:	**IVM_M1S_det**	0.055	0.174	0.016	0.404	0.648	0.352
	IVM_M2S	0.042	0.063	0.003	0.071	0.178	0.822
	IVM_MBASE_bar	0.099	0.031	0.104	0.406	0.639	0.361

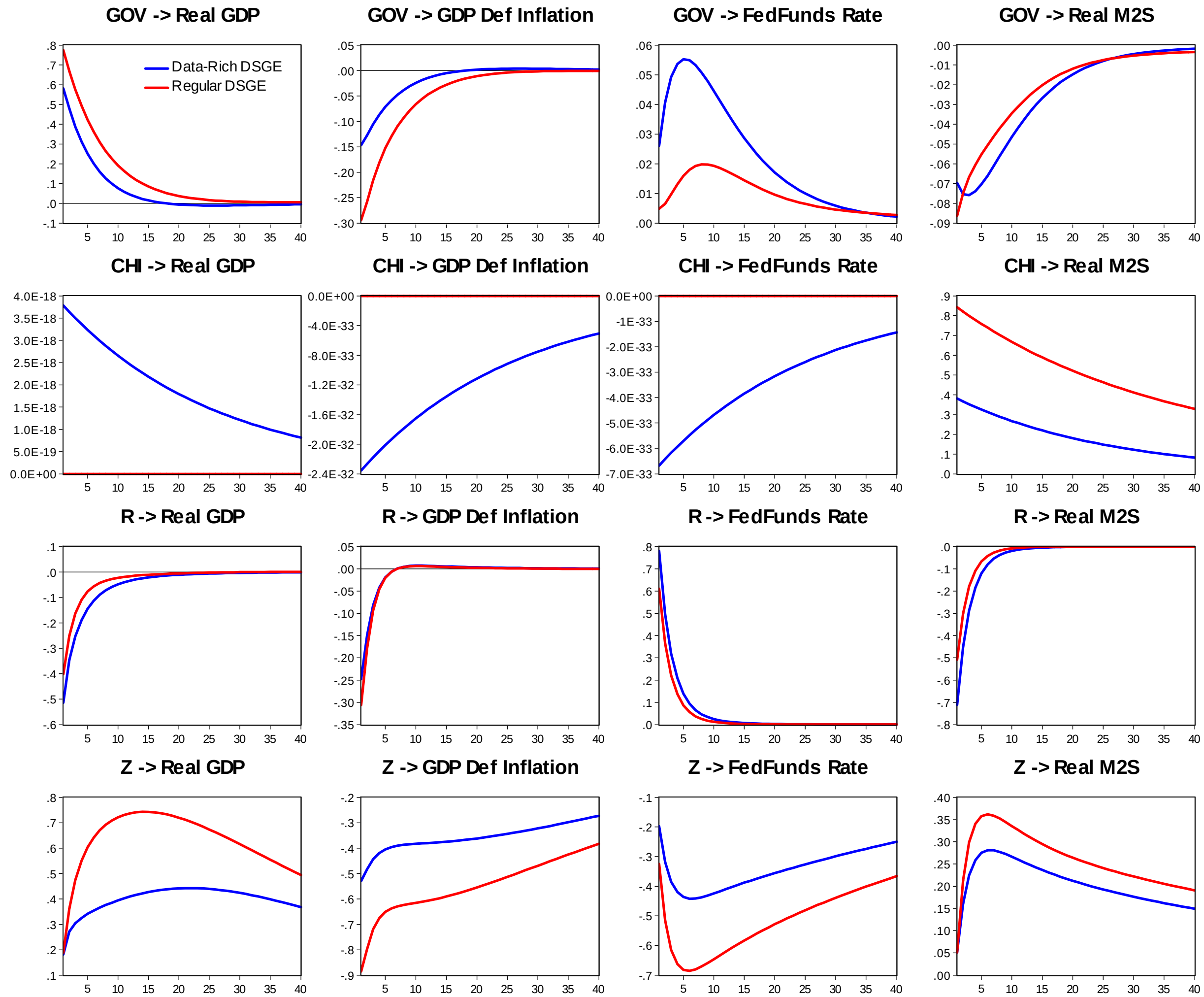
Notes: Structural shocks are GOV - government spending, CHI - money demand, MP - monetary policy, and Z - neutral technology.
Data-Rich DSGE Model: *iid* errors; dataset = dfm3.txt; algorithm: Jungbacker-Koopman; 20K draws, 4K burn-in; VD: posterior mean
Regular DSGE Model: no measurement errors; dataset = 4 primary observables; 100K draws, 20K burn-in; VD: posterior mean

Figure D1. Data-Rich DSGE Model (iid errors): Estimated Model States



Notes: Figure depicts the posterior means and 90% credible intervals of the data-rich DSGE model state variables (blue line and bands): inflation (PI_T, π_t), nominal interest rate (R_T, R_t), real consumption (X_T, x_t), government spending shock (GOV_T, g_t), money demand shock (CHI_T, χ_t), and neutral technology shock (Z_T, Z_t). Red line corresponds to the smoothed versions of the same variables in a *regular* DSGE model estimation derived by Kalman smoother at posterior mean of deep structural parameters (see notes to Table D3 for definition of "regular DSGE estimation").

Figure D2. Impulse Responses to Structural Shocks: Primary Observables



Shocks: GOV - government spending; CHI - money demand; R - monetary policy; Z - technology

Figure D3. Impact of Monetary Policy Innovation on Core Macro Series: Regular vs. Data-Rich DSGE Model

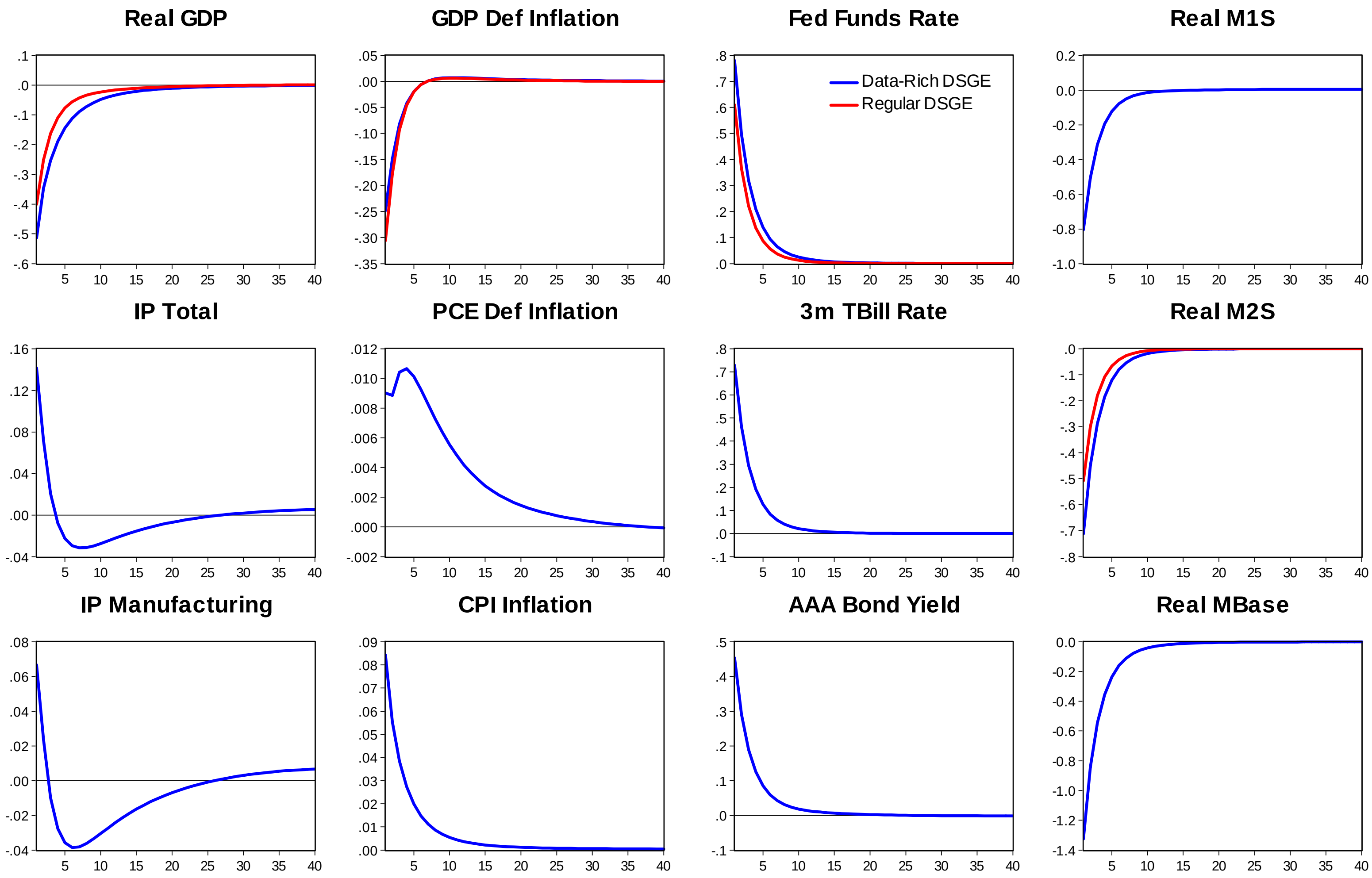
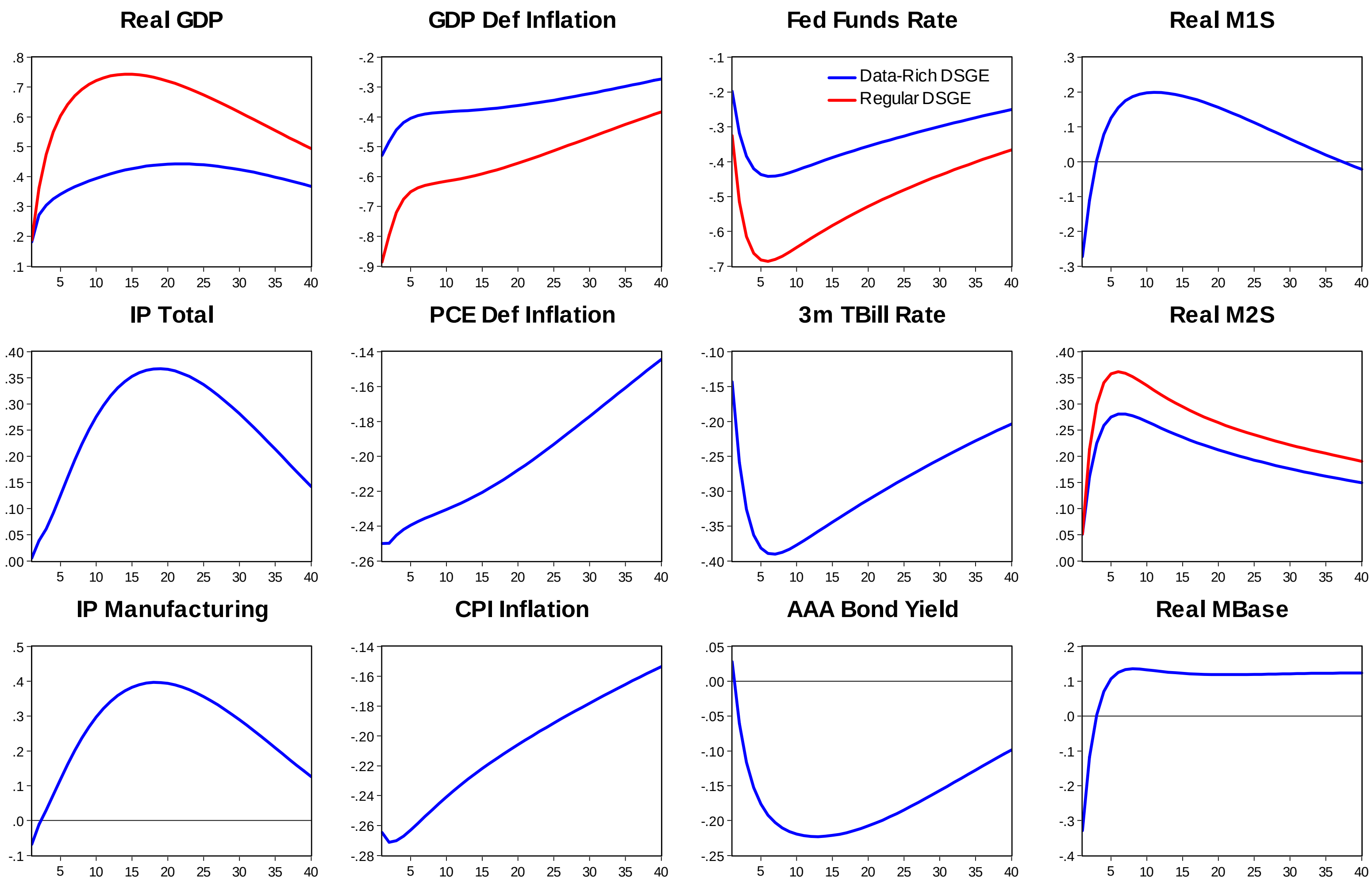


Figure D4. Impact of Technology Innovation on Core Macro Series: Regular vs. Data-Rich DSGE Model



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